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The Photodisintegration of the Lightest Nuclei.

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THE PHOTODISINTEGRATION OF THE LIGHTEST NUCLEI

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ABSTRACT

The sum rule calculations of Levinger and Bethe for dipole transitions in the Nuclear Photoeffect assuming two body central forces of Majorana type have been generalized to include two body central plus tensor forces containing Majorana and Heisenberg type exchange operators, but the effects of retardation have been neglected In Chapter II, explicit expressions for the dipole bremsstrahlung weighted cross section $\sigma_b = \int \frac{\sigma}{W} dW$ and the cross section integrated over photon energy $\sigma_{int} = \int \sigma dW$ have been derived.

In Chapter III, the formulae developed in Chapter II, have been applied to calculate σ_b and σ_{int} for the deuteron photoeffect and comparisons with Levinger's calculations and with experiments have been made. Expressions for σ_b and σ_{int} for the magnetic dipole transitions have been derived from first principles An analytic result is derived for the electric dipole contribution to the deuteron photoeffect cross section, for a central Serber force containing an r^{-2} repulsive core and has been compared with the Bethe-Peierls calculations.

In Chapter IV, expressions for σ_b and σ_{int} have been derived for H^3 and He^3 using Irving's wave function for the three body system and assuming only two body central Heisenberg plus Majorana exchange forces. On assuming charge independence it is found that $\sigma_b(H^3) = \sigma_b(He^3)$ and $\sigma_{int}(H^3) = \sigma_{int}(He^3)$. No comparison with experiments has been made due to lack of experimental data

In Chapter V, σ_b and σ_{int} are calculated for Helium using Irving's wave function for tensor force. The results are compared with values found from the experiments of deSaussure and Osborne, Smith and Barton, Fuller, and Halpern et al. The low theoretical value for σ_b may be related to the small root mean square radius of the alpha particle given by Irving's wave function.

CHAPTER I

INTRODUCTION

In the theory of atomic structure, we deal with electric properties of nucleus and electrons, and we therefore know the forces acting between them. The problem of atomic physics has therefore not been to determine the forces between atomic particles but to find out how electrons move if subjected to a known force. Such a problem has been solved in principle by applying quantum theory.

On the other hand, in nuclear theory, though we have confidence that the quantum theory holds, we do not know the forces between the nuclear particles with the exception of the Coulomb repulsion between the protons in the nucleus. There are two ways in which one can obtain a consistent picture of the structure of the atomic nucleus. One of these is the study of the elementary particles, their properties and mutual interactions, and their interaction with the electromagnetic field. The other way consists in gaining, by direct experimentation, as many data as possible for individual nuclei, and examining the relation among these data. One expects to obtain a network of correlations and connections, which indicate some knowledge of nuclear structure and nuclear forces. These two ways have not yet met to establish a complete understanding of the nucleus, although many connections have been found.

The notion that nuclear forces can be described by a two body interaction, has led to a detailed study of the two body problem theoretically as well as experimentally. Several physicists have carried out experiments on nucleon-nucleon scattering and deduced phase shifts as functions of energy from these scattering experiments, but so far it has not been possible to derive these phase shifts from a well defined potential. It is therefore hard to apply this knowledge to nuclei which are so different from free and isolated nucleons.

A theoretical study of the photodisintegration of the deuteron has been carried out by many authors, e.g., Bethe and Peierls (35), Levinger (49), Schiff (50) Marshall and Guth (50), Austern (53) and it is found that the theory gives a reasonable description of the experimental results for γ-rays of energies below 20 Mev. Although the electromagnetic interaction between field and charge current is completely known, yet in the case of nuclei heavier than the deuteron, there is no theory of comparable exactness. This is because of the complex character of the charge-current vector within nuclei, which turns out to depend on the fine detail of nucleon motion. Even if the basic nuclear interactions were known, the many body problem is beset with very great mathematical difficulties.

The problem of unraveling the structural and dynamical properties of nuclear systems through the study of their interaction with electromagnetic radiation has proved difficult for two reasons. Firstly, the electromagnetic interactions of nucleons are inseparable from their mesonic interactions, and as we are not in possession of a satisfactory meson theory, we do not know the precise form of the interaction of a nuclear system with the electromagnetic field. Secondly, we do not have very satisfactory models of nuclear structure on which we can base quantitative calculations. Quantum mechanically speaking, we lack reasonably good wave functions to represent the initial and final states of the nuclear system needed to calculate radiative transition probabilities and also lack a knowledge of the proper interaction Hamiltonian to be employed in calculating the matrix elements for such transitions. The interaction problem was first studied by Siegert (37). Siegert showed that for dipole transitions and large photon wave-lengths

$$p_{on} = im\, \omega_{on} \int \psi_n^* \sum_i z_i \, \psi_o \, d\tau$$

Here p is the component of the momentum and z is the component of the displacement along the electric vector of the electromagnetic field. ω_{on} is the difference of energies $E_n - E_o$ divided by $\hbar$.

Siegert's work has been generalized and based on the principle of gauge invariance by Sachs and Austern (51) and more recently by Foldy (53). The argument runs that space exchange forces between neutrons and protons imply the existence of a current of charged particles between the nucleons. The complete calculation of the dipole matrix element should involve the sum of nucleon and pion currents. But the nucleons are the source of the pion current, which makes it possible to relate this total current to the nucleon position alone, giving $\sum_i z_i$ for the sum over all protons.

Sachs and Austern showed that this same argument applied to all electric multipole matrix elements in the long wavelength limit, where the photon interacts with the total current, without separating it into nucleonic and mesonic components. However, exchange currents give small contributions to magnetic multipole matrix elements. Foldy has considered the possibility that the nucleon charge distribution should be regarded as spread out over a small volume and depends on coordinates of pairs of nucleons, creating a modification of Siegert's theorem even in the long wavelength limit. Until meson-theoretic calculations can be made correctly, one must use a phenomenological theory for nuclei. A phenomenological theory cannot be rigorously justified, so in the following, we shall take the simplest available, i.e., Siegert's theorem, as generalized by Sachs to all electric multipoles. We would like to mention at this point that it has been shown by Møller and Rosenfeld (43) that the contributions of virtual mesons to the electric dipole and electric quadrupole moment operators of the two nucleon systems are zero through terms of order $v_{nucleon}/c$. A similar elimination of the meson field from the magnetic dipole transitions cannot be made, since it is the meson charge density and not the meson current density that is approximately zero.

Two methods have been used to calculate the photon absorption cross section for nuclei. First, the dipole matrix element can be calculated for absorption of photons of various energies if explicit assumptions are made as to both the ground state wave function and the excited state wave function.

A second method makes use of sum rules, and involves knowledge only of the nuclear wave function for the ground state. It may seem surprising that one wave function alone should be sufficient to determine very much about the photon absorption cross section. But we can argue as follows: the ground state wave function is a solution of Schrödinger's equation and this together with the energy eigenvalue, determines the potential. Schrödinger's equation could then in principle be solved, for this potential for the wave functions of the various excited states. Thus the sum rule involves no new physical assumptions. However, potentials that do not commute with position will in general be different for the excited states than for the ground state. We can still use the sum rule method but the results are changed by an amount proportional to the strength of the exchange force.

To explain the maximum cross sections at similar energies in photodisintegration processes of nuclei, Goldhaber and Teller (1948) postulated an ordered dipole vibration in the nucleus. It was assumed that the γ-rays excite a motion in which the bulk of the protons move in one direction while the neutrons move in the other. Such a vibration would have a high frequency as the protons would have to be separated from the neutrons to which they are tightly bound. The breadth of the resonances was considered as due to the transfer of energy from the ordered vibration by coupling with other modes of nuclear motion, i.e., a process analogous to damping by friction.

Several modes can be set up but the calculations were made for the case where the protons and neutrons are assumed to behave as two interpenetrating, incompressible fluids suffering relative displacement, so that at the nuclear surface, they no longer overlap. The total restoring force is then proportional to the surface area (i.e. R^2).

Making the additional simplifying assumption that $N = Z = \frac{A}{2}$, the resonance frequency is found to be.

$$\hbar\omega_0 = 40\ A^{-1/6} \quad \text{Mev.}$$

and the integrated total cross section for nuclear γ-ray absorption

$$\int_0^\infty \sigma\, dW = 0.015\ A \quad \text{Mev.-barn}$$

Without making any specific assumptions about a nuclear model, Feenberg (36), Siegert (37) and Levinger and Bethe (1950) have calculated the total integrated cross section for photon absorption by applying the sum rule for the matrix elements.

For ordinary forces, the integrated total cross section for electric dipole absorption is:

$$\int_0^\infty \sigma\, dW = 0.060\ \frac{NZ}{A} \quad \text{Mev. -barn}$$

which for $N = Z = \frac{A}{2}$ reduces to Goldhaber-Teller result 0.015 A Mev.-barn. Feenberg (36) and Siegert (37) have shown that the sum rules are modified for Majorana exchange forces, and Levinger and Bethe put in the fraction of the neutron proton exchange force (x) as a parameter in the calculation. Then, using the Hartree approximation of a single nucleon moving in the potential due to the average positions of all others (i.e., assuming no correlation effects between nucleons), neglecting surface effects and considering the case of pure central forces only, they calculate the results from the dipole sum rules for both square and Yukawa wells for the neutron-proton potential.

For a square well, a nuclear radius of $r = 1.5\, A^{1/3} f$ ($f = 10^{-13}$cm) and including the effect of the exchange operator with interference, the result obtained is:

$$\int_0^\infty \sigma\, dW = 0.060\ \frac{NZ}{A}\ (1 + 0.8\, x)\ \text{Mev.-barn}$$

The high energy n-p scattering experiments (Christian and Hart (1950)) indicate a value for x of about 1/2; though the saturation of nuclear forces and shell structure in light nuclei gives x = 0.8 (Inglis (53)).

In Chapter II, we have developed the basic formulae for electric dipole, electric quadrupole and magnetic dipole transitions from first principles and have extended the work of Levinger and Bethe to include non-central forces of the Yukawa type. These non-central forces also include Bartlett and Heisenberg exchange operators as well as the Majorana exchange operator considered by Levinger and Bethe.

In Chapter III, we have applied the formulae developed in Chapter II to the deuteron problem and compared the results with experiments. We find that the tensor forces have very little effect on the dipole bremsstrahlung weighted cross section $\sigma_b = \int \frac{\sigma}{W}\, dW$, though they increase the integrated cross section $\sigma_{int} = \int \sigma \, dW$ by about 7 percent. We derive σ_b^{M-1} and σ_{int}^{M-1} (M-1 and E-1 denote magnetic and electric dipoles respectively) from first principles for central forces, and show that for the non-central forces σ_b^{M-1} does not change. In the last section of this chapter, we calculate an analytic expression for the electric dipole contribution to the deuteron photoeffect cross section, for a central Serber force containing an r^{-2} repulsive core. The form of the wave function corresponding to this potential is also given and has been used to calculate σ_b^{E-1} and σ_{int}^{E-1}.

In Chapter IV, we calculate the above mentioned two moments for the three body problem using spin dependent central forces and Irving's (51) wave function. We find that due to charge independence

$$\sigma_b^{E-1}(H^3) = \sigma_b^{E-1}(He^3)$$
$$\text{and } \sigma_{int}^{E-1}(H^3) = \sigma_{int}^{E-1}(He^3).$$

Chapter V contains an evaluation of σ_b^{E-1} and σ_{int}^{E-1} for the helium nucleus and a comparison with experiments. We have calculated these quantities using Irving's (51, 53) wave functions for central and non-central forces. We have compared the size of the alpha particle obtained from these wave functions with the experimental value of Hofstadter (56) for the root mean square (rms) radius and find that the rms radius is much smaller than the experimental value. We have as a solution suggested the inclusion of a repulsive core in the wave function to increase the rms radius. We have left the result for σ_{int}^{E-1} in terms of x and y (the fractions of Majorana and Heisenberg exchange forces).

CHAPTER II

BASIC FORMALISM

The Oscillator Strength

The cross section for the absorption of photons by bound charged particles can be calculated treating the photons as a classical electromagnetic field and applying time-dependent perturbation theory. Such a theory will account for absorption, but not for emission. We shall first present the calculation for electric dipole absorption from ground state 'O' to excited discrete state 'n' (see for example Mott and Sneddon, Sec. 46.1).

The probability $\left|C_n(t)\right|^2$ that after time 't' the system has made a transition from state 'O' to state 'n', is

$$\left|C_n(t)\right|^2 = \left| -\frac{i}{\hbar} \int_0^t e^{i\omega_{on}t'} \left[H'(r,t')\right]_{on} dt' \right|^2 \tag{2.1}$$

Here the angular frequency $\omega_{on} = (E_n - E_o)/\hbar$; and the perturbation H' due to a spatially constant electric field of amplitude E acting on a particle along the positive z axis is

$$H'(r,t') = -eEz \cos\omega t' . \tag{2.2}$$

The matrix element $\left[H'(r,t')\right]_{on}$ is found by spatial integration.

$$\left[H'(r,t')\right]_{on} = -eE\cos\omega t' \; z_{on} = -eE\cos\omega t' \int \psi_o^* z \psi_n d^3r \tag{2.3}$$

where ψ_o is the atomic wave function for the state 'O' and ψ_n for the state 'n'.

Inserting the time dependence of H' into Eqn. (2.1) and carrying out the integration, we get

$$\left| C_n(t) \right|^2 = e^2 E^2 \left| z_{on} \right|^2 \left[\frac{\sin^2 (\omega_{on} - \omega)\, t/2}{\hbar^2 \, (\omega_{on} - \omega)^2} + \frac{\sin^2 (\omega_{on} + \omega) t/2}{\hbar^2 \, (\omega_{on} + \omega)^2} \right] \tag{2.4}$$

At resonance only the first term in the parenthesis is of importance, but it gives the transition rate to state 'n' as proportional to 't'. In order to obtain a sensible expression for the probability of finding the electron in excited state'n', we integrate the probability amplitude over a line of spectrum of 'ω' and obtain

$$\int \left| c_n(t) \right|^2 d\omega = e^2 E^2 \left| z_{on} \right|^2 \pi t / 2\hbar^2 \tag{2.5}$$

which gives the transition rate independent of 't' as expected.

We now introduce the integrated absorption cross section σ_{on} for a single transition in an atom.

$$\int \sigma_{on} \, d\omega = \frac{\int (\text{transition/sec}) \; d\omega}{\text{photon}/\text{cm}^2 \ \text{sec}} \tag{2.6}$$

We evaluate the denominator from the classical Poynting vector $c \, \dfrac{\vec{E} \times \vec{H}}{4\pi}$ with $\vec{E} = \vec{H}$ in Gaussian units and dividing it by the photon energy $\hbar \omega_{on}$ to give the photon flux as $c \, E^2/8\pi\hbar\omega_{on}$ (the factor 1/2 comes from a spatial average). Then

$$\int \sigma_{on} \, d\omega = \frac{\frac{1}{t} \int |c_n(t)|^2 \, d\omega}{c \, E^2 / 8\pi\hbar\omega_{on}} = \frac{4\pi^2 e^2 \hbar \, \omega_{on} \, |z_{on}|^2}{\hbar^2 \, c} \tag{2.7}$$

We further introduce the mean oscillator strength f_{on} as

$$f_{on} = \frac{2 \, m \, \omega_{on}}{\hbar} \left| z_{on} \right|^2 = \left| \frac{z_{on}}{\lambda\!\!\!^{-}_{on}} \right|^2 \tag{2.8}$$

Here z is the component of the displacement along the direction of polarization of the photon; and, $\lambda\!\!\!^{-}_{on}$ is the wavelength divided by 2π

for a particle of mass 'm' and energy $(E_n - E_o)$.

If the initial state of our system is an 's' state, the mean oscillator strength f_{on} is the same for any polarization direction of the electromagnetic field: $|Z_{on}|^2 = |X_{on}|^2 = |Y_{on}|^2 = \frac{1}{3} \, |r_{on}|^2$. We include all allowed 'n' values for the final state. If the initial state has $l \neq o$, we must average over the 'm' values of the initial state.

From the last two equations

$$\int \sigma_{on} \, d\omega = \frac{2\pi^2 e^2}{mc} \, f_{on} \tag{2.9}$$

The oscillator strength f_{on} is convenient to use for three reasons: it is dimensionless; the summed oscillator strength is unity; and, it provides connections between quantum mechanical and classical calculations for the interaction of a charged oscillator with an electric field.

If we are dealing with absorption leading to continuum states at energy $\hbar\omega = W$ above the ground state, we rewrite Eqn. (2.9) as

$$\sigma(W) = \frac{2\pi^2 e^2 \hbar}{mc} \, \frac{df}{dW} \tag{2.10}$$

The Oscillator density per unit energy $\frac{df}{dW}$ can be calculated from Eqn. (2.8). The matrix element Z_{on} used in calculating df/dW is found using a continuum wave function.

<u>The Thomas-Reiche-Kuhn (TRK) sum-rule</u>

The Thomas-Reiche-Kuhn sum-rule states that for an electron in any potential $V(r)$, the summed oscillator strength $\sum_n f_{on}$ is unity. Summing means: sum over discrete final states and integrate over the continuum. From Eqn. (2.8) we write

$$\sum_n f_{on} = \left(\frac{2m}{\hbar^2}\right) \sum_n (E_n - E_o)\,(z_{on})^*\,(z_{on})$$

$$= \frac{m}{\hbar^2} \sum_n \left[(E_n - E_o)\, z_{on} z_{no} + z_{on} (E_n - E_o)\, z_{no} \right] \qquad (2.11)$$

We use the matrix relations

$$(E_n - E_o)\, z_{on} = - \left[H, Z\right]_{on}$$

$$(E_n - E_o)\, z_{no} = \left[H, z\right]_{no} \qquad (2.12)$$

where the square bracket is the commutator of the Hamiltonian operator

and z. Thus we have

$$\sum_n f_{on} = \frac{m}{\hbar^2} \sum_n \left\{ - \left[H, z\right]_{on} z_{no} + z_{on} \left[H, z\right]_{no} \right\} \qquad (2.13)$$

We thus obtain using closure

$$\sum_n f_{on} = - \frac{m}{\hbar^2} \left[\left[H, z\right], z\right]_{oo}$$

$$= \frac{m}{\hbar^2} \left\{ \left[\left[p^2/2m, z\right], z\right] + \left[\left[V(r), z\right], Z\right] \right\}_{oo} \qquad (2.14)$$

Using $\left[p_z, z\right] = - i\hbar$ and $\left[V(r), z\right] = o$, we obtain

$$\sum_n f_{on} = 1.$$

The above derivation shows that the TRK sum rule holds rigorously provided that the potential 'V' commutes with the coordinate 'z'.

Many electron problem

In this section, we shall calculate E-1 oscillator strength in a more elegant manner, discuss applications to the many electron problem, and find the expressions for two multipoles E-2 and M-1.

In Eqn. (2.2) we took the perturbation due to the electromagnetic field as $\vec{E} \cdot \vec{r}$. Here we shall start with a more fundamental expression in which the vector potential $\vec{A} = A_0\, \vec{u}\, e^{-i\vec{k}\cdot\vec{r}}\, e^{i\omega t}$ is written into the atomic Hamiltonian by replacing the kinetic energy term $\frac{p^2}{2m}$ by

$\frac{(\vec{p} - e\vec{A}/c)^2}{2m}$. $\vec{u}$ is a unit vector along the polarization direction and $\vec{k}$ is the photon wave number. In our first order perturbation theory treatment, we discard the A^2 term, the perturbation being

$$H'_1(\vec{r}, t) = -\frac{e}{mc}\vec{A}.\vec{p}$$
$$= -\left(\frac{eA_o}{mc}\right)\vec{u}.\vec{p}\ e^{-i\vec{k}.\vec{r}}\ e^{i\omega t} \tag{2.15}$$

(We use the gauge where div $\vec{A}$ = o). The E-1 non-retarded approximation consists in putting $e^{-i\vec{k}.\vec{r}}$ = unity. This approximation is justifiable in a non-relativistic system of spatial extent about equal to R, since $\vec{k}.\vec{r} \simeq v/c \ll 1$

The oscillator strength f_{on} is proportional to the square of the matrix element of the perturbation.

$$f_{on} \sim \left|(H')_{on}\right|^2 \sim \left|(\vec{u}.\vec{p})_{on}\right|^2 \sim (m\,\omega_{on})^2 \left|(\vec{u}.\vec{r})_{on}\right|^2 \tag{2.16}$$

where we have used

$$(\vec{u}.\vec{p})_{on} \sim \frac{m(E_n - E_o)}{\hbar}(\vec{u}.\vec{r})_{on} = m\,\omega_{on}(\vec{u}.\vec{r})_{on} \tag{2.17}$$

We can show that the factors of m and ω_{on} in (2.17) agree with those in (2.8) by using the relation $\vec{E} = \frac{1}{c}\frac{\partial\vec{A}}{\partial t}$.

The generalized TRK sum rule $\sum_n f_{on}$ = Z holds rigorously as we can see from its derivation in Eqn. (2.14). We replace z for a single electron by $\sum_i z_i$ for all Z electrons in the atom.

$$\sum_n f_{on} = -(m/\hbar^2)\left[\left[H, \sum_i z_i\right], \sum_i z_i\right] = \sum_{i=1}^{Z} 1 = Z \tag{2.18}$$

Here $H = \sum_i \frac{p_i^2}{2m} + V(\vec{r}_1, \vec{r}_2, ---, \vec{r}_Z)$ with the commutators $\left[p_{z_i}, z_{i'}\right] = -i\hbar\delta_{ii'}$ and $\left[V, z_i\right] = 0$

So far we have confined our attention to the allowed electric dipole (E-1) transitions and we have neglected retardation. We shall now derive expressions for the electric quadrupole (E-2) and magnetic dipole (M-1) transitions following Sachs (53) (See Sachs, Blatt and Weisskopf, Foldy (53) and Stech for detailed treatment of multipole expansions). We have already used the first term of unity in the expansion of exp $(-i\,\vec{k}.\vec{r})$ in Eqn. (2. 15) while writing H_1'. We now take the term $-i(\vec{k}.\vec{r})$ in the expansion of the exponential, and write the additional perturbation as

$$\begin{aligned} H_2' &= -\frac{eA_o}{mc}\ (\vec{u}.\vec{p})\ (-i\,\vec{k}.\vec{r}) \\ &= -\left(\frac{ieA_o}{2mc}\right)\left\{\left[(\vec{k}.\vec{r})(\vec{u}.\vec{p}) + (\vec{u}.\vec{r})(\vec{k}.\vec{p})\right]\right. \\ &\qquad \left. +\left[(k.r)(u.p) - (u.r)(k.p)\right]\right\} \end{aligned} \tag{2. 19}$$

The first square bracket on the right gives E-2 transitions. We again use $\vec{p} = m\,\dot{\vec{r}}$ and write

$$1/2\ \left[(\vec{k}.\vec{r})(\vec{u}.\vec{p}) + (\vec{u}.\vec{r})(\vec{k}.\vec{p})\right] = 1/2\ m\ \frac{d}{dt}\left[(\vec{k}.\vec{r})(\vec{u}.\vec{r})\right] \tag{2.20}$$

This expression replaces $(\vec{u}.\vec{p}) = m\ \frac{d}{dt}(\vec{u}.\vec{r})$ in the E-1 matrix element. Comparing with Eqn. (2.8) we find the E-2 oscillator strength as

$$f^Q_{on} = \frac{m\omega}{2\hbar}\left\{\left[(\vec{k}.\vec{r})(\vec{u}.\vec{r})\right]_{on}\right\}^2 \tag{2.21}$$

f^Q_{on} is fairly small and has not been calculated for any of the cases discussed in this thesis.

Magnetic dipole (M-1) transitions are due to the second square bracket in Eqn. (2. 19) which we write

$$\begin{aligned} &-(i\,eA_o/2mc)\ \left[(\vec{k}.\vec{r})(\vec{u}.\vec{p}) - (\vec{u}.\vec{r})(\vec{k}.\vec{p})\right] \\ &= -(i\,eA_o/2mc)\left[(\vec{k}\times\vec{u})\ .\ (\vec{r}\times\vec{p})\right] \\ &= -(i\,eA_o/2mc)\ \ k\,L_H \end{aligned} \tag{2. 22}$$

Here the orbital angular momentum $\vec{L} = \vec{r} \times \vec{p}$. L_H is the component of $\vec{L}$ along the photon's magnetic field, i.e., in the direction perpendicular both to propagation direction $\vec{k}$ and polarization direction $\vec{u}$. The M-1 oscillator strength is

$$f_{on}^{M-1} = \frac{\omega}{2mc^2\hbar} \left[(L_H)_{on} \right]^2 \tag{2.23}$$

We shall discuss the sum rule for f^{M-1} for the deuteron using anomalous magnetic moments in Chapter III.

E-2 absorption from an 'S' state leads to a D state with $m = \pm 1$. M-1 absorption is important in the nuclear case in the ^3S to ^1S photomagnetic disintegration of the deuteron.

<u>The Nuclear Photoeffect</u>

The nuclear photoeffect is more complicated compared to the atomic photoeffect for two reasons. First, the nucleons all have comparable mass, while in the atomic photoeffect we considered the electrons as bound in an infinitely heavy atom. Second, there are exchange forces between pairs of nucleons; so the two body potential does not commute with the relative coordinates.

Throughout the thesis, we shall adopt a phenomenological approach to the nuclear photoeffect. We shall calculate the interaction of the nucleus with the electromagnetic field in terms of nucleon coordinates alone. We shall neglect the effects of the mesons at low energies, though we can not deny the possibility that mesons must be considered explicitly even at low energies. We shall assume that nucleons inside a free nucleus have the same properties as in free space though we know that nucleons inside a nucleus might possibly have different charges, masses and, magnetic moments (Johnson and Teller, 55).

Calculations of the photoelectric cross section for a nuclear transition from the ground state 'O' to a particular excited state demands knowledge of the wave functions of both the ground state and excited state. Very little is known of the wave function for the ground state of nuclei and much less is known of the wave functions for the excited state. It is for this reason that we shall sum over all excited states, using closure for the matrix elements, so that our results will depend only on the wave function assumed for the ground state.

We express the E-1 interaction of the spatially constant electromagnetic field with the nucleons in the z direction as:

$$\begin{aligned} H' &= e\,E \sum_i z_i \\ &= e\,E\left[\frac{N}{A}\sum_i z_i - \frac{Z}{A}\sum_j z_j + \frac{Z}{A}\sum_i z_i + \frac{Z}{A}\sum_j z_j\right] \\ &= e\,E\left[\frac{N}{A}\sum_i z_i - \frac{Z}{A}\sum_j z_j + Z\,\bar{z}\right] \end{aligned} \tag{2.24}$$

where 'i' stands for the proton, 'j' for the neutron and $\bar{z}$ denotes the position of the centre of mass of the nucleus. The term $(eEZ\bar{z})$ corresponds to interaction of the entire nucleus with the electric field and leads to nuclear Thomson scattering. The term $eE\left[\frac{N}{A}\sum_i z_i - \frac{Z}{A}\sum_j z_j\right]$ gives photon absorption due to internal motion in the nucleus That is for E-1 absorption, each proton acts as if it has an effective charge $e\frac{N}{A}$ (See. Bethe 37).

The TRK sum rule is therefore modified to the form:

$$\sum_n f_{on} = \left\{ \left(\frac{N}{A}\right)^2 \sum_i \left[p_{z_i} , z_i \right] + \left(-\frac{Z}{A}\right)^2 \sum_j \left[p_{z_j} , z_j \right] \right\} \left(\frac{i}{\hbar}\right)$$
$$= \frac{N^2}{A^2} Z + \frac{Z^2}{A^2} N = \frac{NZ}{A} \tag{2.25}$$

This result is given in Levinger and Bethe (1950).

From the above Eqn. we see that $\sum_n f_{on} = o$ whenever N = o or Z = o. In a direct photoneutron emission, the rest of the nucleus is accelerated away from tne neutron, leaving the neutron free with the momentum it had in the nuclear ground state.

In the electron photoeffect, we can write the matrix element for interaction of the charges with the electric field $\vec{E}$ as $(\vec{E}.\vec{D})_{on}$ or as $(\vec{j}.\vec{A})_{on}$ where $\vec{D}$ is the electric dipole moment, $\vec{j}$ the current and $\vec{A}$, the vector potential. We can transform from one expression to the other using (for a single charged particle).

$$(j_z)_{on} = \frac{e}{M} (p_z)_{on} = i\, e\, \omega_{on} (z)_{on}. \tag{2.26}$$

The relation $(P_z)_{on} = m\, i\, \omega_{on} (z)_{on}$ does not hold if the potential energy term 'V' in the Hamiltonian fails to commute with the position. We have instead.

$$i\, \omega_{on} (z)_{on} = \frac{i}{\hbar} \left[\frac{P^2}{2m} , z \right]_{on} + \frac{i}{\hbar} \left[V, z \right]_{on}. \tag{2.27}$$

Since Eqn. (2.26) does not hold, we are uncertain whether $\vec{E}.\vec{D}$ or $\vec{j}.\vec{A}$ should be used in calculations of the nuclear photoeffect. (See Breit and Condon, 36). An answer to this question was given by Seigert. Seigert's theorem shows that we should use the $i\, \omega_{on} (z)_{on}$ matrix element. Siegert's original argument was made more rigorous and generalized to all electric multipoles by Sachs and Austern (51) and by Foldy (53). As this topic is slightly out of our present discussion, we shall not go through the proof of Siegert's theorem. We might remark that Siegert's theorem fails at high photon energies where the interaction $\vec{j}.\vec{A}$ involves the detailed structure of the current $\vec{j}(r)$ (See Brennan and Sachs, 52).

We shall now derive the TRK sum rule for a single particle subject to exchange potential of the type $V = J(r_{ij})\ P^M_{ij}$, where P^M_{ij} is the Majorana exchange operator. Here we are interested in evaluating the double commutator $-\ \frac{M}{\hbar^2}\left[\left[\ V,\ z_i\right],\ z_i\right]_{oo}$

The commutator

$$\left[V,\ z_i\right] = J(r_{ij})\ P^M_{ij}\ z_i - z_i\ J(r_{ij})\ P^M_{ij} = (z_j - z_i)\ J(r_{ij})P^M_{ij}\ . \tag{2.28}$$

Similarly we can show that

$$-\ \frac{M}{\hbar^2}\left[\left[V,\ z_i\right],\ z_i\right]_{oo} = -\ \frac{M}{\hbar^2}\left[(Z_i - Z_j)^2\ J(r_{ij})\ P^M_{ij}\right]_{oo} \tag{2.29}$$

$$= -\ \frac{M}{3\hbar^2}\left[r^2_{ij}\ \ J(r_{ij})\ P^M_{ij}\right]_{oo} \quad \text{(for a spherical nucleus)}$$

Therefore

$$\sum_n f_{on} = -\ \frac{M}{\hbar^2}\left[\left[H,\ z_i\right],\ z_i\right]_{oo}$$
$$= 1 - \frac{M}{3\hbar^2}\left[r^2_{ij}\ \ J(r_{ij})\ P^M_{ij}\right]_{oo} \tag{2.30}$$

which tells us that an attractive Majorana potential increases the E-1 sum. The above sum rule was first derived by Feenberg (36) and Siegert (37).

In the following, from the summed oscillator strength, we shall first find $\int \sigma\ dW$, where W is the photon energy, and σ is the cross section for photon absorption (σ is the sum of all the partial cross sections for the various nuclear reactions that may occur subsequent to the photon absorption: γ-n, γ -p, γ-γ etc.). We shall later on derive an expression for $\int \frac{\sigma}{W}\, dW$ (referred to as σ_b on account of its resemblance with the bremsstrahlung spectrum).

From Eqn. (2.25) and (2.9) we find that for a nucleus made up of N neutrons and Z protons

$$\int \sigma\, dW = \frac{2\pi^2 e^2 \hbar}{Mc}\ \frac{NZ}{A} \quad . \tag{2.31}$$

Thus for the case $N = Z = \frac{A}{2}$

$$\int \sigma\, dW = \frac{\pi^2 e^2 \hbar}{Mc}\ \frac{A}{2} = 0.015A \text{ Mev.-barn} \tag{2.32}$$

This result agrees with that given by Goldhaber and Teller (48) and also by Levinger and Bethe (50).

Feenberg and Siegert have shown that the sum rules are modified for exchange forces. We shall therefore consider the effect of Majorana type exchange forces on σ_{int}.

From Eqns. (2.9), (2.14), (2.24), (2.25), it follows that for a nucleus with 'N' neutrons and 'Z' protons

$$\int \sigma\, dW = \frac{2\pi^2 e^2 \hbar}{Mc} \left\{ \frac{NZ}{A} - \frac{Mx}{4\hbar^2} \left[\left[\sum_i \sum_j V(r_{ij}) P^M_{ij} \, , \, \left(\sum_i z_i - \sum_j z_j \right) \right] , \left(\sum_i z_i - \sum_j z_j \right) \right]_{oo} \right\} \tag{2.33}$$

where 'x' is a fraction of the entire force between neutron and proton. We have written $V = x\, V(r_{ij}) P^M_{ij}$ where r_{ij} is the distance between proton and neutron and P^M_{ij} is the operator for exchanging the space co-ordinates of the ith proton with the jth neutron. Using the property that

$$\left[\left[\sum_i \sum_j P^M_{ij}\, ,\, \sum_i z_i\right]\, ,\, \sum_i z_i\right] = \left[\sum_i \sum_j (z_j - z_i)\, P^M_{ij}\, ,\, \sum_i z_i\right]$$

$$= \sum_i \sum_j (z_j - z_i)^2\, P^M_{ij} \tag{2.34}$$

we obtain from (2.33)

$$\int \sigma\, dW = \frac{2\pi^2 e^2 \hbar}{Mc}\left\{\frac{NZ}{A} - \frac{Mx}{\hbar^2}\left[\sum_i \sum_j (z_j - z_i)^2\, V(r_{ij}) P^M_{ij}\right]_{oo}\right\} \tag{2.35}$$

which for a spherical nucleus takes the form

$$\int \sigma\, dW = \frac{2\pi^2 e^2 \hbar}{Mc}\left\{\frac{NZ}{A} - \frac{Mx}{3\hbar^2}\left[\sum_i \sum_j r_{ij}^2\; V(r_{ij})\, P^M_{ij}\right]_{oo}\right\} \tag{2.36}$$

which is in agreement with Levinger and Bethe (50) and Siegert (37). This formula can be used for He^4 without any modification because He^4 is an even-even nucleus.

The introduction of a tensor interaction gives a term of the form $\sum_i \sum_j P^M_{ij} S_{ij} J(r_{ij})$ where $J(r_{ij})$ defines the shape of the well and

$$S_{ij} = \frac{3\,(\vec{\sigma}_i \cdot \vec{r}_{ij})\,(\vec{\sigma}_j \cdot \vec{r}_{ij})}{r_{ij}^2} - (\vec{\sigma}_i \cdot \vec{\sigma}_j) \tag{2.37}$$

In the presence of central and tensor forces

$$\int \sigma\, dW = \frac{2\pi^2 e^2 \hbar}{Mc}\left\{\frac{NZ}{A} - \frac{1}{4}\frac{Mx}{\hbar^2}\left[\left[\sum_i \sum_j V(r_{ij})\, P^M_{ij}\, ,\, \left(\sum_i z_i - \sum_j z_j\right)\right],\, \left(\sum_i \;{}_i - \sum_j z_j\right)\right]_{oo} - \frac{1}{4}\frac{Mx'}{\hbar^2}\left[\left[\sum_i \sum_j J(r_{ij}) P^M_{ij}\, S_{ij},\, \left(\sum_i z_i - \sum_j z_j\right)\right],\, \left(\sum_i z_i - \sum_j z_j\right)\right]_{oo}\right\} \tag{2.38}$$

where the tensor Majorana force is a fraction x' of the entire force between neutron and proton. Since S_{ij} commutes with the space coordinate, we have

$$\left[\left[\sum_i \sum_j P^M_{ij}\ S_{ij}, \sum_i z_i\right], \sum_i z_i\right] = \sum_i \sum_j (z_i - z_j)^2\ P^M_{ij}\ S_{ij}. \tag{2.39}$$

Therefore

$$\int \sigma\, dW = \frac{2\pi^2 e^2 \hbar}{Mc}\left\{ \frac{NZ}{A} - \frac{Mx}{3\hbar^2}\left[\sum_i \sum_j V(r_{ij})\ P^M_{ij}\ r^2_{ij}\right]_{oo} - \frac{Mx'}{3\hbar^2}\left[\sum_i \sum_j r^2_{ij}\ J(r_{ij})\ S_{ij}\ P^M_{ij}\right]_{oo}\right\} \tag{2.40}$$

which is similar to Eqn. (2.34).

The Bartlett operator exchanges the spin directions of the two particles, leaving their positions unaffected. Bartlett forces therefore will not contribute anything to σ_{int} because the Bartlett operator commutes with the space coordinates

$$\left[P^B, z_i\right] = o. \tag{2.41}$$

The Heisenberg operator interchanges both position and spin coordinates, therefore

$$\left[P^H_{ij}, z_i\right] = \left[P^B_{ij}\ P^M_{ij}, z_i\right] = (z_j - z_i)\ P^B_{ij}\ P^M_{ij} \tag{2.42}$$

and for central forces of the form $V(r_{ij})\left[1 + xP^M_{ij} + yP^H_{ij}\right]$ (where 1 includes Wigner and Bartlett forces and x and y denote the fractions of Majorana and Heisenberg type forces).

$$dW = \frac{2\pi^2 e^2 \hbar}{Mc}\left\{ \frac{NZ}{A} - \frac{Mx}{3\hbar^2}\left[\sum_i \sum_j V(r_{ij})\ r^2_{ij}\ P^M_{ij}\right]_{oo} - \frac{My}{3\hbar^2}\left[\sum_i \sum_j V(r_{ij})\ r^2_{ij}\ P^H_{ij}\right]_{oo}\right\} \tag{2.43}$$

For a large even-even nucleus, any pair of neutrons and protons will have a probability of 3/4 for being in a spin triplet state and 1/4 for being in a spin singlet state. Since the Bartlett operator gives + 1 when it acts on a spin triplet state and -1 acting on a spin singlet state, therefore

$$\left[\psi_o^* \; P^H \psi_o \right] = \frac{1}{2} \left[\psi_o^* \; P^M \psi_o \right] \tag{2.44}$$

where ψ_o is the spatial wave function of the nucleus and we have averaged over the nucleon spins.

According to Rosenfeld (48) the most satisfactory mixture from saturation requirements may be written as

$$0.93\; P_{ij}^M - 0.13 - 0.26\; P_{ij}^H + 0.46\; P_{ij}^B \tag{2.45}$$

A simplified version that is almost equivalent to this in many rough calculations, because it keeps the large terms and contains the same proportion of terms involving spin exchange to those which do not, has been suggested by Inglis (50). According to Inglis, the mixture may be taken as

$$0.8\; P_{ij}^M + 0.2\; P_{ij}^B\,. \tag{2.46}$$

A third mixture which explains the n-p scattering data at high energies has been given by Serber (47). According to Serber, a satisfactory mixture may be written as

$$0.5 + 0.5\; P_{ij}^M \tag{2.47}$$

Lavinger and Bethe (1950) have calculated the integrated cross section σ_{int} for a mixture of Wigner and Majorana neutron proton forces. They find that for an even-even nucleus

$$\sigma_{int} = \int \sigma\, dW = 15A\,(1 + 0.8x)\ \text{Mev.-mb.} \tag{2.48}$$

If in addition to Majorana exchange we also include Heisenberg exchange force, then using (2.44) we find

$$\sigma_{int} = 15A\,(1 + 0.8x + 0.4y)\ \text{Mev.-mb} \tag{2.49}$$

For Rosenfeld mixture (x = .93 and y = -.26)

$$\sigma_{int} = 15\,(1.64)A = 24.60\ \text{A Mev.-mb.} \tag{2.50}$$

One gets the same result for Inglis mixture. For Serber mixture (x = .5, y = 0)

$$\sigma_{int} = 15A\,(1 + 0.4) = 21A\ \text{Mev.-mb.} \tag{2.51}$$

A most general Yukawa potential including tensor interaction may be written as

$$\begin{aligned} V = &-V_o\left[\,(1 - x - y) + x\,P_{ij}^{M} + y\,P_{ij}^{H}\right] J(r_{ij}) \\ &-\gamma' V_o\left[(1 - x' - y') + x'\,P_{ij}^{M} + y'\,P_{ij}^{H}\right] K(r_{ij})\,S_{ij} \end{aligned} \tag{2.52}$$

where $J(r_{ij}) = \dfrac{e^{-r_{ij}/r_c}}{r_{ij}r_c}$; $K(r_{ij}) = \dfrac{e^{-r_{ij}/r_t}}{r_{ij}/r_t}$; r_c and r_t being the ranges of central and tensor potentials. x and x', y and y' denote fractions of central and tensor forces respectively of Majorana and Heisenberg type. The potential $V_o(1 - x - y)$ includes both Wigner and Bartlett interactions. In (2.52) x and y in general have different numerical values from the x and y in (2.43) for pure central forces. There is not much information available on the numerical values of x, y, x' and y' for use in Eqn. (2.52). P-P scattering experiments at energies about 100 Mev. have indicated that $x' \neq 1/2$. This follows from the strong polarization due to tensor force acting in 'P' state (Rochester Conference 1956).

For a potential of the type (2.52)

$$\int \sigma\,dW = \frac{2\pi^2 e^2 \hbar}{Mc}\left\{\frac{NZ}{A} - \frac{Mx}{3\hbar^2}\left[\sum_i \sum_j V_o\,J(r_{ij})r_{ij}^2\,P_{ij}^{M}\right]_{oo} - \right.$$

$$- \frac{My}{3\hbar^2}\left[\sum_i \sum_j V_o J(r_{ij})r_{ij}^2\,P_{ij}^{H}\right]_{oo} -$$

$$- \frac{\gamma' M x'}{3 \hbar^2} \left[\sum_i \sum_j V_o K(r_{ij}) S_{ij} r_{ij}^2 P_{ij}^M \right]_{oo}$$

$$- \frac{\gamma' M y'}{3 \hbar^2} \left[\sum_i \sum_j V_o K(r_{ij}) r_{ij}^2 S_{ij} P_{ij}^H \right]_{oo} \Bigg\} \tag{2.53}$$

We shall now obtain an expression for $\sigma_b = \int \frac{\sigma}{W} \, dW$. From (2.8), (2.9) and (2.24), we can write

$$\sigma_b = \int \frac{\sigma}{W} \, dW = \frac{2\pi^2 e^2 \hbar}{Mc} \sum_n \frac{f_{on}}{E_n - E_o}$$

$$= \frac{2\pi^2 e^2 \hbar}{Mc} \cdot \frac{2M}{\hbar^2} \sum_n \left| \frac{N}{A} \sum_i (z_i)_{on} - \frac{Z}{A} \sum_j (z_j)_{on} \right|^2$$

$$= 4\pi^2 \left(\frac{e^2}{\hbar c}\right) \frac{NZ}{A} (z^2)_{oo} = \frac{4\pi^2}{3} \left(\frac{e^2}{\hbar c}\right) \frac{NZ}{A} \langle r^2 \rangle_{oo} \tag{2.54}$$

For a single proton

$$\sum_n f_{on} / (E_n - E_o) = \sum_n (2M/\hbar^2) \, |z_{on}|^2 = \frac{2M}{\hbar^2} \langle z^2 \rangle_{oo}$$

$$\sigma_b = \frac{4\pi^2}{3} \frac{e^2}{\hbar c} \langle r^2 \rangle_{oo} \tag{2.55}$$

Expressions (2.54) and (2.55) have been previously derived by Levinger and Bethe (50). The above result is independent of whether the force between nucleons is ordinary or exchange in character. Tensor forces also do not change the above expressions.

CHAPTER III

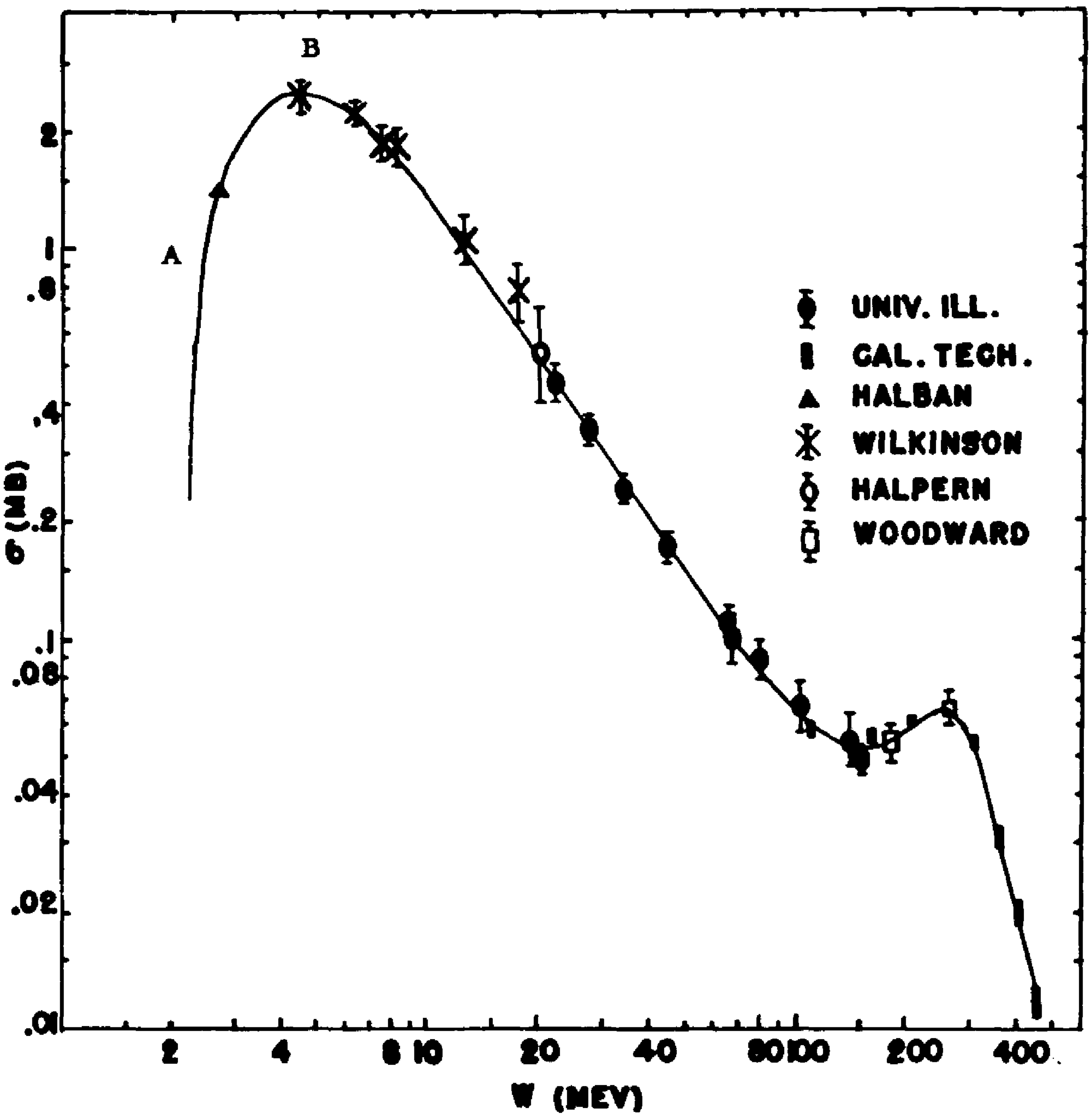
APPLICATIONS TO DEUTERON

In this chapter we shall confine ourselves to the photodisintegration of the deuteron. We shall summarize the experimental results and discuss the sum rule calculations of $\sigma(W)$ in detail. We shall calculate the electric and magnetic dipole bremsstrahlung weighted cross section and also σ_{int} with central and tensor forces.

Fig. 1 shows a plot of the total cross section (measured in mb = 10^{-27} cm^2) vs. W, the photon energy in Mev. The small bump 'A' in the curve is due to the photomagnetic transitions ${}^3S_1 \rightarrow {}^1S_0$ and ${}^3D_1 \rightarrow {}^1D_2$. The main peak 'B' is due to E-1 transitions. The smooth curve is the result of the measurements made in various laboratories by Keck et al (54a, 54b), Barnes (86), Halpern (53), Whalin (54), etc.

A sum rule calculation for the photodisintegration of the deuteron has been made by Levinger (55) and Levinger and Bethe (50). In his paper, Levinger has calculated the bremsstrahlung weighted cross section σ_b, the integrated cross section σ_{int} and $\overline{W}\ \sigma_{int}$ and has compared the results with the experiments.

In the present calculation, we shall calculate σ_b and σ_{int} for E-1 transitions using two body repulsive core wave function and repulsive core potential due to Rustgi and Levinger (such wave function had been



A
B
2
1
.8
.4
.2
.1
.08
.04
.02
.01
σ (MB)
2
4
8 10
20
40
80 100
200
400
W (MEV)
UNIV. ILL.
CAL. TECH.
HALBAN
WILKINSON
HALPERN
WOODWARD

used by Austern - private communication) and also study the effect of the tensor forces using the wave functions of Feshbach, Schwinger and Harr (49).

The bremsstrahlung weighted cross section is proportional to the average value of r^2 (See Eqn. 2.55) for the deuteron ground state and is independent of the fraction 'x' of the neutron-proton force that has an exchange character. We shall give $\langle r^2 \rangle_{oo}$ first for zero range n-p force, then using effective range theory and finally give the small modification introduced by using a Hulthén wave function. For zero range $\langle r^2 \rangle_{oo}$ is $(2\gamma^2)^{-1}$ where γ equals $\left(\frac{M\epsilon}{\hbar^2}\right)^{1/2}$ with ϵ the deuteron binding energy. Since the inside wave function gives only a small contribution, we can calculate $\langle r^2 \rangle_{oo}$ using the outside wave function of effective range theory, introducing a factor of $(1-\gamma r_o)^{-1}$ for the squared normalization constant of the outside wave function, as in the Bethe-Longmire (50) effective range correction to the Bethe-Peierls cross section.

Applying Eqn. (2.54) for the deuteron, we get

$$\sigma_b^{E-1} = \frac{\pi^2 e^2 \hbar}{6Mc} \quad (1-\gamma r_o)^{-1} \tag{3.1}$$

Use of a Hulthén wave function

$$\psi_o = \frac{N}{r} \left(e^{-\gamma r} - e^{-\beta r} \right) \tag{3.2}$$

gives $(r^2)_{oo}$ about three percent smaller than the effective range calculation (3.1). The bremsstrahlung weighted cross section σ_b for electric dipole absorption is given by

$$\sigma_b^{E-1} = \frac{\pi^2 e^2 \hbar}{6Mc} \quad (1-\gamma r_o)^{-1} \quad (0.97). \tag{3.3}$$

The three factors in this equation correspond, respectively, to the zero range value, the effective range correction and the small correction to effective range theory using Hulthén wave function (See Levinger (55)).

If we use repulsive core wave function due to Rustgi and Levinger and Austern (unpublished)

$$\psi = \sqrt{\frac{2\gamma}{1-\gamma r_o}} \quad \frac{e^{-\gamma r} - e^{-\beta' r} - (\beta' - \gamma) r e^{-\beta' r}}{r} \tag{3.4}$$

(where β' is again determined from effective range theory and is found to be $\beta' = 11.565\gamma$ for $r_o(-\epsilon, -\epsilon) = 1.66$ f) and evaluate $(r^2)_{oo}$, we find

$$\langle r^2 \rangle_{oo} = \frac{(1-\gamma r_o)^{-1}}{2\gamma^2}\left[1 + \frac{1}{(\beta'/\gamma)^3} + \frac{3(\beta'/\gamma - 1)^2}{(\beta'/\gamma)^5} - \frac{16}{(1+\beta'/\gamma)^3} - 48\frac{(\beta'/\gamma - 1)}{(1+\beta'/\gamma)^4} + \frac{3(\beta'/\gamma - 1)}{(\beta'/\gamma)^4}\right]. \tag{3.5}$$

Therefore

$$\frac{\sigma_b^{E-1}(R.C.)}{\sigma_b^{E-1}(\text{Hulthén})} = \frac{1}{0.97}\left[1 + \frac{1}{(\beta'/\gamma)^3} + \frac{3(\beta'/\gamma - 1)^2}{(\beta'/\gamma)^5} - \frac{16}{(1+\beta'/\gamma)^3} - \frac{48(\beta'/\gamma - 1)}{(1+\beta'/\gamma)^4} + \frac{3(\beta'/\gamma - 1)}{(\beta'/\gamma)^4}\right] \tag{3.6}$$

$$= 1.00$$

In other words the bremsstrahlung weighted cross section is not changed due to the introduction of the repulsive core.

The discovery of the quadrupole moment of the deuteron required the introduction of non-central forces into the phenomenological description of the neutron-proton interaction. Following the notation of Feshbach and Schwinger (1951), we find that

$$\frac{\sigma_b^{E-1}(\text{tensor})}{\sigma_b^{E-1}(\text{Hulthén})} = \rho_o^2\left[\frac{2\gamma^2(1-\gamma r_Q)}{0.97}\right] \frac{\int_0^\infty (u^2+w^2)x^2 dx}{\int_0^\infty (u^2+w^2)\, dx} =$$

$$= \frac{(7.6176)(2011)}{(1051)} \left[\frac{2\gamma^2(1-\gamma r_0)}{0.97} \right]$$

Where $x = r/\rho_0$; $\rho_0 = 2.76$ f and the integrals on the right have been evaluated numerically using the numerical values of the wave functions from Feshbach et al (49)

$$\sigma_b^{E-1}(\text{Tensor}) \Big/ \sigma_b^{E-1}(\text{Hulthén}) = 0.994 \qquad (3.7)$$

This indicates that the tensor forces do not change the bremsstrahlung weighted cross section compared to central forces by any significant amount.

Since the electric dipole bremsstrahlung weighted cross section can be calculated with high accuracy, it is worthwhile to include an estimate of the magnetic dipole contribution which we designate σ_b^{M-1} (Electric quadrupole and retardation effects are smaller and tend to cancel). Levinger (55) has evaluated σ_b^{M-1} integrating the effective range theory cross section and finds that

$$\sigma_b^{M-1} = \left(\frac{\pi^2 e^2}{3\hbar c}\right)(\mu_p - \mu_n)^2 \left(\frac{\hbar}{Mc}\right) = 0.23 \text{ mb.} \qquad (3.8)$$

Here μ_p and μ_n are the proton and neutron magnetic moments (this result is independent of the singlet scattering length). While constants in Eqn. (3.8) are known with high accuracy, it is unclear how well this calculation applies to the deuteron, as we have neglected the exchange contribution due to meson currents.

In the present discussion, we shall derive Eqn. (3.8) from first principles, first for central and later on for tensor forces. We shall show that Eqn. (3.8) does not change on introducing tensor forces.

For the sake of simplicity let us first treat the ground state of the deuteron as a pure $^3S_1$ state. A magnetic dipole transition can then only produce a ^1S_o state. From Eqn. (2.23) applied to the deuteron, we find that

$$\sum_n \frac{f_{on}^{M-1}}{(E_n-E_o)} = \frac{1}{2Mc^2\hbar^2} \sum_n \left| (\vec{\nu}\cdot\vec{M})_{on} \right|^2 . \tag{3.9}$$

where $\vec{\nu}$ is the unit vector normal to the direction of polarization and to the direction of propagation of the photon. The vector $\vec{M}$ is just the magnetic moment operator of the deuteron given by

$$\vec{M} = \hbar\left[\frac{\vec{L}}{2} + \mu_p\vec{\sigma}_p + \mu_n\vec{\sigma}_n\right]$$

$$= \hbar\left[\frac{\vec{L}}{2} + (\mu_p + \mu_n)\vec{S} + \frac{1}{2}(\mu_p - \mu_n)(\vec{\sigma}_p - \vec{\sigma}_n)\right]. \tag{3.10}$$

Since both the initial and final states are 'S' functions, the matrix element of $\vec{L}$ vanishes. Furthermore the vector $\vec{S} = \frac{1}{2}(\vec{\sigma}_p + \vec{\sigma}_n)$ acting on the single spin function χ_o vanishes ($\vec{S}\chi_o = o$). The matrix element of $\vec{S}$ also vanishes between any two $^3S_1$ states since the radial functions being solutions of the same Schrödinger equation for different energies are orthogonal. Therefore only the last term contributes to the transition, which can just lead to a singlet triplet transition, since it is antisymmetric for interchange of the neutron and proton. The initial state is

$$\psi_i = A' \frac{u_i}{r} \chi_1^m \tag{3.11}$$

where χ_1^m is the triplet spin function, and the final state is

$$_f = A' \frac{u_f}{r} \chi_o \tag{3.12}$$

where χ_o is the singlet spin function.

The spin matrix element, when averaged over all initial states of polarization of the deuteron gives

$$\frac{1}{3}\,(\chi_1^{\,m}\,,\ \vec{\nu}\cdot(\vec{\sigma}_p-\vec{\sigma}_n)\chi_o)\,(\chi_o,(\vec{\nu}\cdot\vec{\sigma}_p-\vec{\sigma}_n)\chi_1^{\,m})\,. \tag{3.13}$$

On using the relation $\vec{\sigma}_n\chi_o = -\vec{\sigma}_p\chi_o$ we get for the spin matrix eler ient

$$\begin{aligned}
&\frac{4}{3}\,(\chi_1^{\,m}\,,(\vec{\nu}\cdot\vec{\sigma}_p)\chi_o)\,(\chi_o,\ (\vec{\nu}\cdot\vec{\sigma}_p)\chi_1^{\,m})\\
&=\ \frac{4}{3}\,(\chi_1^{\,m}\ (\vec{\nu}\cdot\vec{\sigma}_p)^2\,\chi_1^{\,m})\\
&=\ \frac{4}{3}
\end{aligned} \tag{3.14}$$

Substituting this in (3.9), we obtain using $\sum_f\ (1)_{if}\,(1)_{if} = (1)_{ii} = 1$ for spatial matrix elements,

$$\sum_n \frac{f_{on}^{\,M-1}}{E_n - E_o} \ = \ \frac{\hbar}{6\,Mc^2\,\hbar^2}\ (\mu_p - \mu_n)^2\ . \tag{3.15}$$

Therefore

$$\begin{aligned}
\sigma_b^{\,M-1} \ &= \ \frac{2\,\pi^2 e^2\hbar}{Mc}\ \ \frac{\hbar^2}{6Mc^2\,\hbar^2}\ \ (\mu_p - \mu_n)^2\\
&= (\pi^2 e^2/3\hbar c)\,(\hbar/Mc)^2\,(\mu_p - \mu_n)^2
\end{aligned} \tag{3.16}$$

which agrees with Levinger (55)

We shall now calculate $\sigma_b^{\,M-1}$ using both S and D wave functions of the deuteron. Since the observed angular momentum of the deuteron is J = 1, the following final states can be reached due to a magnetic dipole transition ($\Delta J = \pm 1$, o.)

J = 1	${}^3S_1{}^*$ ${}^3D_1{}^*$ ${}^1P_1$
J = o	${}^1S_o$ ${}^3P_o$
J = 2	${}^1D_2$, ${}^3P_2$ and ${}^3F_2$.

The matrix elements of transitions ${}^3S_1 \longrightarrow {}^3S_1{}^*$ or ${}^3D_1 \longrightarrow {}^3D_1{}^*$ vanish since the radial functions being solutions of the same Schrödinger Equation for different energies are orthogonal. From parity considerations it can be shown that the matrix elements for all other transitions vanish except for two namely ${}^3S_1 \longrightarrow {}^1S_o$ and ${}^3D_1 \longrightarrow {}^1D_2$. Since the 'D' transition is not large in itself and since it does not interfere with the 'S' transition in total cross section, we expect the 'D' contribution to be fairly small. Let

$$\psi_i = \frac{1}{r}\left[u(r) + \frac{w(r)}{\sqrt{8}} S_{12} \right]\chi_m \tag{3.17}$$

and

$$\psi_f = \frac{1}{r}\left[u(r) + \frac{w'(r)}{\sqrt{8}} S_{12} \right]\chi_o \tag{3.18}$$

We are neglecting the contribution of transitions due to $\frac{\vec{L}}{2}$ term in (3.10) because it is very small. In order to evaluate σ_b^{M-1}, we find that the wave function in this case has the same spin dependence as for the central case and therefore

$$\sum_n \frac{f_{on}^{M-1}}{E_n - E_o} = \frac{\hbar}{6Mc^2 \hbar^2} (\mu_p - \mu_n)^2 \frac{\left[\int_o^\infty u^2\, dr + \frac{1}{8} \int_o^\infty \chi_1^m S_{12}^2 \chi_1^m w^2\, dr\right]}{\int_o^\infty (u^2 + w^2)\, dr} \tag{3.19}$$

On using $S_{12}^2 \chi_1^m = (8 - 2 S_{12})\chi_1^m$, we get

$$\sum_n \frac{f_{on}^{M-1}}{E_n - E_o} = \frac{\hbar}{6Mc^2 \hbar^2} (\mu_p - \mu_n)^2 \tag{3.20}$$

and therefore

$$\sigma_b^{M-1} = \left(\frac{\pi^2 e^2}{3\hbar c}\right) \left(\hbar/_{Mc}\right)^2 \left(\mu_p - \mu_n\right)^2 \tag{3.21}$$

Thus σ_b^{M-1} is not changed because of the tensor forces.

We shall now calculate $\sigma_{int} = \int \sigma dW$ for central (Hulthén and repulsive core) and tensor forces. Levinger and Bethe have already calculated σ_{int} for Hulthén wave function and Yukawa potential well.

<u>Hulthén wave function and central Hulthén potential</u>

Hulthén wave function may be written as

$$\psi_o = \sqrt{\frac{2\gamma}{1-\gamma r_o}} \left[\frac{e^{-\gamma r} - e^{-\beta r}}{r} \right] \tag{3.22}$$

Substitution into the wave equation shows that this wave function corresponds to the triplet interaction potential $V(r)$ given by

$$V(r) = -\frac{\hbar^2}{M} \left(\beta^2 - \gamma^2\right) \frac{e^{-\beta r}}{e^{-\gamma r} - e^{-\beta r}}$$

$$= -V_o \frac{e^{-\beta r}}{e^{-\gamma r} - e^{-\beta r}}$$

where $$V_o = \frac{\hbar^2}{M} \left(\beta^2 - \gamma^2\right). \tag{3.23}$$

Since P^H and P^M both give +1 operating on a ground ^3S state, therefore from Eqn. (2.43) we have

$$\int \sigma dW = \frac{\pi^2 e^2 \hbar}{Mc} \left[1 - \frac{2}{3} \frac{M(x+y)}{\hbar^2} \int \psi_o^* \; Vr^2 \psi_o \; d\tau \right] \tag{3.24}$$

where x and y are defined in (2.52).

But $$\int \psi_o^* \; Vr^2 \psi_o \, r^2 \, dr = -\frac{2\gamma V_o}{(1-\gamma r_o)} \left[\frac{2}{(\gamma+\beta)^3} - \frac{1}{4\beta^3} \right] \tag{3.25}$$

Therefore

$$\frac{2}{3} \frac{M}{\hbar^2} \int \psi_o^* \; Vr^2 \psi_o \; r^2 dr = \frac{4}{3} \frac{(\beta^2/\gamma^2 - 1)}{(1-\gamma r_o)} \left[\frac{2}{(1+\beta/\gamma)^3} - \frac{1}{4\beta^3} \right] \tag{3.26}$$

Using $(\beta/\gamma) = 6.395$ for $r_o(-\epsilon, -\epsilon) = 1.66$ f (3.27)

$$\frac{2}{3}\frac{M}{\hbar^2}\int \psi_o^* \, V \, \psi_o \, r^4 dr = 0.345 \tag{3.28}$$

and therefore

$$\sigma_{int}^{E-1}\,(\text{Hulthén}) = \frac{\pi^2 e^2 \hbar}{Mc}\left[1 + 0.345\,(x+y)\right]$$
$$= 30\left[1 + 0.345\,(x+y)\right] \text{ Mev.-mb} \tag{3.29}$$

The value $r_o(-\epsilon, -\epsilon) = 1.66$ f in the above calculation has been taken to make a comparison with the tensor force calculation but for comparison with experiment, $r_o(-\epsilon, -\epsilon) = 1.76$ f is most suitable (Salpeter, 51). A calculation with $r_o = 1.76$ f gives a 7 percent increase in the coefficient 0.345.

<u>Repulsive core potential and wave function.</u>

A substitution of the repulsive core wave function

$$\psi_o = \sqrt{\frac{2\gamma}{1-\gamma r_o}}\;\frac{\left[e^{-\gamma r} - e^{-\beta' r} - (\beta' - \gamma)\,r\,e^{-\beta' r}\right]}{r} \tag{3.30}$$

into the wave equation gives the following triplet interaction potential

$$V(r) = -V_o\,\frac{\left[-1 + (\beta' + \gamma)\,r\right]e^{-\beta' r}}{e^{-\gamma r} - e^{-\beta' r} - (\beta' - \gamma)\,r\,e^{-\beta' r}} \tag{3.31}$$

where $\beta' = 11.5658\,\gamma$ (3.32)

(from effective range theory for $r_o(-\epsilon, -\epsilon) = 1.66$ f)

and $$V_o = \frac{\hbar^2}{M}(\beta - \gamma)^2 \tag{3.33}$$

We find in this case that

$$-\frac{2}{3}\frac{M}{\hbar^2}\int \psi_o^* \, V r^2 \, \psi_o \, r^2 \, dr$$
$$= \frac{4}{3}\frac{(\beta'/\gamma - 1)^2}{(1-\gamma r_o)}\left[\frac{4}{(1+\beta'/\gamma)^3} - \frac{1}{2(\beta'/\gamma)^3} - \frac{3}{4}\frac{1}{(\beta'/\gamma)^4}\right.$$
$$\left. + \frac{3}{4}\frac{1}{(\beta'/\gamma)^5}\right] = 0.400. \tag{3.34}$$

Therefore

$$\sigma_{int} = 30 \left[1 + 0.4 (x + y) \right] \text{ Mev.-mb.} \tag{3.35}$$

The increase of σ_{int} in this case over (3.28) has been caused due to larger value of $\sigma_{R.C.}$ compared to $\sigma_{Hulthén}$ up to energies ~ 50 Mev.

σ_{int} with tensor force

The general form of the interaction potential here is taken to be

$$V(r) = - V_o \left\{ \left[(1 - \frac{g}{2}) + \frac{g}{2} (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \right] \frac{e^{-r/r_c}}{r/r_c} + \gamma' S_{12} \frac{e^{-r/r_t}}{r/r_t} \right\} \tag{3.36}$$

Here 'r' is the interparticle distance; r_c and r_t are the ranges of the potentials for the central and tensor potentials. V_o, γ', and g are constants; and, we have assumed the following values of the constants for the present calculation which are almost but not exactly equal to the Pease-Feshbach values. (We cannot use the exact Pease-Feshbach values because we need tabulated values of the wave functions from Feshbach et al (49).)

$$\begin{aligned} r_c &= 1.185 \text{ f} \quad ; \quad r_t = 1.533 \text{ f} \\ \gamma' &= 0.782 \quad ; \quad \rho_o = 2.76 \text{ f} \end{aligned} \tag{3.37}$$

In order to evaluate $\langle Vr^2 \rangle_{oo}$, we shall follow the method of Blatt and Weisskopf (52). We shall first write S_{12} in a slightly different form. If we take $\vec{S} = \frac{1}{2} (\vec{\sigma}_1 + \vec{\sigma}_2)$, then we can show that

$$S_{12} = \frac{2 \left[3 (\vec{S} \cdot \vec{r})^2 - S^2 r^2 \right]}{r^2} \tag{3.38}$$

But for a deuteron S = 1, therefore,

$$S^2 = S(S + 1) = 2 \tag{3.39}$$

and $$S_{12} = 6 \left[\vec{S} \cdot \frac{\vec{r}}{r} \right]^2 - 4. \tag{3.40}$$

Let us now write

$$\psi = \frac{1}{r}\left[u(r)\,\mathcal{Y}^{M}_{101} + w(r)\,\mathcal{Y}^{M}_{121}\right] \tag{3.41}$$

where $\mathcal{Y}^{M}_{JlS}$ is the normalised spin angle wave function belonging to a state of total angular momentum J whose Z component is 'M' and which is a combination of an orbital angular momentum '1' with a spin 'S'. From Blatt and Weisskopf (52).

$$\mathcal{Y}^{M}_{JlS} = \sum_{M} C_{lS}(J, M; m_l, m_s)\, Y_{lm_l}(\theta, \varphi)\, \chi_{Sm_s} \tag{3.42}$$

For the deuteron S = 1 and J = M = S = 1; 1 = o, 2 and thus we have

$$\mathcal{Y}^{1}_{101} = \left(\frac{1}{4\pi}\right)^{1/2} \chi_{1,1} = Y_{oo}\,\chi_{11} \tag{3.43}$$

$$\mathcal{Y}^{1}_{121} = \left(\frac{6}{10}\right)^{1/2} Y_{2,2}\,\chi_{1,-1} - \left(\frac{3}{10}\right)^{1/2} Y_{2,1}\,\chi_{1,0} + + \left(\frac{1}{10}\right)^{1/2} Y_{2,0}\,\chi_{1,1} \tag{3.44}$$

We shall follow the method of Blatt and Weisskopf to find the effect of the tensor operator on the spin angle wave functions $\mathcal{Y}$. We find that

$$(\vec{S}\cdot\frac{\vec{r}}{r})^2\,\mathcal{Y}^{M}_{101} = \frac{2}{3}\,\mathcal{Y}^{M}_{101} + \frac{\sqrt{2}}{3}\,\mathcal{Y}^{M}_{121} \tag{3.45}$$

$$\text{and } (\vec{S}\cdot\frac{\vec{r}}{r})^2\,\mathcal{Y}^{M}_{121} = \frac{1}{3}\,\mathcal{Y}^{M}_{121} + \frac{\sqrt{2}}{3}\,\mathcal{Y}^{M}_{101} \tag{3.46}$$

Using the orthogonal properties of spin angle functions, we find that

$$\left[\int_0^\infty (u^2+w^2)d\rho\right]\langle Vr^2\rangle_{oo} = -V_o\,\rho_o^{\,2}\left[\frac{(1-g)(x+y)}{\beta}\int_0^\infty u^2 e^{-\beta\rho}\rho\, d\rho + \frac{(x+y)}{\beta}\int_0^\infty w^2 e^{-\beta\rho}\rho\, d\rho + \frac{4\sqrt{2}\gamma'(x'+y')}{\tau}\int_0^\infty uwe\, e^{-\tau\rho}\rho\, d\rho - \frac{2\gamma(x'+y')}{\tau}\int_0^\infty w^2 e^{-\tau\rho}\rho\, d\rho\right] \tag{3.47}$$

where $$\rho = \frac{r}{\rho_o} \quad ; \quad \beta = \frac{\rho_o}{r_c} \quad ; \quad \tau = \frac{\rho_o}{r_t} \tag{3.48}$$

Evaluating the integrals numerically, we get

$$\langle Vr^2 \rangle_{oo} = - V_o \rho_o^2 \frac{\left[40.37(x + y) + 1.82(x + y) + 72.82(x'+y') - 5.43(x'+y') \right]}{1051}$$

$$= - V_o \rho_o^2 \left[0.04 \, (x + y) + 0.064 \, (x'+y') \right] . \tag{3.49}$$

Therefore

$$\sigma_{int}^{E-1} \text{(tensor)} = 30 \left[1 + \frac{2}{3} \frac{M}{\hbar^2} V_o \rho_o^2 \left\{ 0.04(x + y) + 0.064(x'+y') \right\} \right]$$

$$= 30 \left[1 + 0.20 \, (x + y) + 0.32 \, (x'+y') \right] \text{Mev.-mb.} \tag{3.50}$$

If we assume $x' + y' = x + y$, the σ_{int} for a tensor force is appreciably larger than that for a central force of corresponding exchange character (See Eqn. (3.28).) For example a Serber central and tensor force of Yukawa type ($x' = x = 1/2$; $y' = y = o$) gives $\sigma_{int}^{E-1} = 30 \, (1.26) = 38$ Mev. -mb, while a Serber central Hulthèn force gives $\sigma_{int}^{E-1} = 30 \, (1.17) = 35$ Mev. -mb. The large contribution of tensor forces comes from the third term in (3.47) where the cross term $u'w'k(\rho)$ is multiplied by the large coefficient $4\sqrt{2}\,\gamma'$. Our result of σ_{int} with tensor forces is in disagreement with Austern (52), who finds only a two percent increase in the central force cross section due to the D term. We shall now calculate σ_{int} for M-1 transition for central spin dependent forces using Hulthèn form of potential. From Eqns. (2.9) and (2.23) we know that

$$dW = \frac{\pi^2 e^2 \hbar}{Mc} \frac{(y + z)}{2Mc^2} \left[\left[V(r) P^H, (\vec{e} \cdot (\mu_p \vec{\sigma}_p + \mu_n \vec{\sigma}_n)) \right], (\vec{e} \cdot (\mu_p \vec{\sigma}_p + \mu_n \vec{\sigma}_n)) \right]_{oo} \tag{3.51}$$

where μ_p and μ_n are the proton and neutron magnetic moments and y and z are fractions of Heisenberg and Bartlett forces respectively.

Now

$$\left[V(r)P^H,\ (\vec{e}.(\mu_p\vec{\sigma}_p + \mu_n\vec{\sigma}_n))\right] = (\mu_p - \mu_n)(\vec{e}.\ \vec{\sigma}_p - \vec{\sigma}_n)\ V(r)P^H \tag{3.52}$$

Therefore

$$\left[\left[V(r)P^H,\ (\vec{e}.(\mu_p\vec{\sigma}_p + \mu_n\vec{\sigma}_n))\right],\ (\vec{e}.(\mu_p\vec{\sigma}_p + \mu_n\vec{\sigma}_n))\right]_{oo}$$

$$= (\mu_p - \mu_n)^2\ \chi_1\ (\vec{e}.\ \vec{\sigma}_n - \vec{\sigma}_p)^2\ \chi_1 \langle V(r)\rangle_{oo} \tag{3.53}$$

But for a deuteron the spin expectation value averaged over the three substates m = 1, 0, -1, gives $\chi_1\left[\vec{e}.(\vec{\sigma}_n - \vec{\sigma}_p)\right]^2\chi_1 = \frac{4}{3}$ and for a Hulthen potential the spatial average gives

$$\langle V(r)\rangle_{oo} = -\frac{2\gamma V_o}{(1-\gamma r_o)}\left[\frac{1}{\gamma+\beta} - \frac{1}{2\beta}\right] \tag{3.54}$$

where

$$V_o = \frac{\hbar^2}{M}(\beta^2 - \gamma^2).$$

Combining all these results, we get

$$\int \sigma\, dW = \frac{\pi^2 e^2 \hbar}{Mc} \cdot \frac{4}{3} \cdot \frac{(y+z)}{2Mc^2}\ \frac{2\gamma}{1-\gamma r_o} \cdot \frac{\hbar^2}{M}\ (\beta^2 - \gamma^2)\left[\frac{1}{(\beta+\gamma)} - \frac{1}{2\beta}\right] \cdot (\mu_p - \mu_n)^2$$

$$= \frac{2\pi^2}{3}\left(\frac{e^2}{\hbar c}\right)(\hbar/Mc)^2\,(y+z)\,(\mu_p - \mu_n)^2\ \frac{\hbar^2}{M} \cdot \frac{2\gamma}{(1-\gamma r_o)}\ (\beta^2 - \gamma^2)\left[\frac{1}{\gamma+\beta} - \frac{1}{2\beta}\right]$$

$$= 7.73\,(y+z)\ \text{Mev.-mb.} \tag{3.55}$$

If y + z = 0.2 (Inglis or Rosenfeld mixture),

$$\int \sigma\, dW = 1.55\ \text{Mev.-mb.} \tag{3.56}$$

An approximate calculation by Levinger (55), and Levinger and Cummings (unpublished) gives

$$\int \sigma\, dW = 1\ \text{Mev.-mb.} \tag{3.57}$$

The following table shows a comparison between the various moments of the experimental cross sections for photodisintegration of the deuteron and the theoretically calculated values using phenomenological sum rules.

	$\sigma_b = \int \sigma \frac{dW}{W}$ in mb	$\sigma_{int} = \int \sigma\, dW$ in Mev.-mb.
1) Experiment	3.9	39
2) Central Hulthen potential	3.50 + .23 = 3.73	$30\left[1 + 0.345\,(x + y)\right] + 1.55$
3) Central Repulsive core potential	3.50 + .23 = 3.72	$30\left[1 + 0.4\,(x + y)\right] + 1.55$
4) Tensor Yukawa potential	3.47 + .23 = 3.70	$30\left[1 + 0.20\,(x+y) + .32\,(x'+y')\right] + + 1.55$

The table shows that the agreement between experiment and theory for the bremsstrahlung weighted cross section is within 5 percent experimental uncertainty (Note. We have used $r_o(-\epsilon, -\epsilon) = 1.66\ f$; a larger r_o gives a larger σ_b). The values for σ_{int} turn out to be different for different mixtures and are tabulated below (using $\sigma_{int}^{M-1} = 1.55$ Mev.-mb).

Mixture	int Central Hulthen Potential	int Central Repulsive Core Pot.	int Tensor Yukawa Potential (assuming $x' = x$; $y' = y$)
Serber	36.7	37.6	39.4
Rosenfeld	38.5	39.6	43.3
Inglis	39.9	41.2	44.0

All these values agree with the experiment within 12 percent. We would at this point like to mention that an $(\vec{L}.\vec{S})$ force does not change σ_{int} or σ_b (Frankel, 54) but velocity dependent potential of the form $V(r)\left[1 + ap^2 + ----\right]$ would change σ_{int} but not σ_b.

We shall now calculate an analytic result for the electric dipole contribution to the deuteron photoeffect cross section $\sigma(W)$ for a central Serber force containing an r^{-2} repulsive core. We have already given the form of the wave function and the corresponding potentials in Eqns. (3.29) and (3.30). We shall first of all calculate the matrix element (M_o^{E-1}) for the electric dipole transition of the deuteron from the ground $^3S_1$ state. Since only the proton has a charge 'e', and since its co-ordinate relative to the centre of gravity of the deuteron is $\frac{\vec{r}}{2}$, we have

$$M_o^{E-1} = \frac{e}{2} \int \psi_o^* \; z \, \psi_f \, d\tau \tag{3.58}$$

if the γ-ray is polarized in the z direction. Here ψ_f is the wave function of the final state normalized per unit energy. From the selection rule for the angular momentum, we infer that the final state ψ_f must be a 'P' state. We know however that moderate energy 'P' states are practically uninfluenced by the short range force between neutron and proton. Also, for a central Serber force, there is no interaction in the final 'P' state. Therefore the wave function for 'P' state will have the same form as if the neutron and proton were free. Following Bethe and Peierls (35), we can write

$$\psi_f = \left(3/4\pi\right)^{1/2} \; \mathrm{Cos}\,\theta \; \left(2/\pi\right)^{1/2} \left(M/2\hbar^2 k\right)^{1/2} \frac{1}{r} \left(-\mathrm{Cos}\,kr + \frac{\mathrm{Sin}\,kr}{kr}\right). \tag{3.59}$$

Or

$$\psi_f = \frac{B \cos\theta}{kr^2} \quad Re \left[e^{ikr}(-i-kr) \right]$$

where $B = \frac{3^{1/2}}{2\pi\hbar} (M/k)^{1/2}$; Re denotes the real part and 'k' represents the wave number in the final state.

Therefore

$$M_o^{E-1} = \frac{eBA'}{2} \; Re \int \frac{e^{-\gamma r} - e^{-\beta' r} - (\beta'-\gamma) r e^{-\beta' r}}{r} \; \frac{r \cos^2\theta}{kr^2} \left[e^{ikr}(-i-kr) \right] \cdot$$

$$\cdot \, 2\pi \sin\theta \, d\theta \; r^2 \; dr$$

$$= \frac{2eM^{1/2}}{3^{1/2}\hbar} \; \frac{k^{3/2}}{(\gamma^2+k^2)^2} \sqrt{\frac{2\gamma}{(1-\gamma r_o)4\pi}} \left[1 - \frac{(\gamma^2+k^2)^2}{(\beta'^2+k^2)^2} - \right.$$

$$\left. - \frac{4\beta'(\beta'-\gamma)(\gamma^2+k^2)^2}{(\beta'^2+k^2)^3} \right] \qquad (3.60)$$

$$R.C^{(W)} = \frac{8\pi^3\nu}{c} \; \left| M_o^{E-1} \right|^2$$

$$= \frac{8\pi}{3} \cdot \frac{e^2}{\hbar c} \cdot \frac{\hbar^2}{M} \; \frac{\epsilon^{1/2} E^{3/2}}{(E+\epsilon)^3} \; \frac{1}{(1-\gamma r_o)}$$

$$\left[1 - \frac{(\gamma^2+k^2)^2}{(\beta'^2+k^2)^2} - \frac{4\beta'(\beta'-\gamma)(\gamma^2+k^2)^2}{(\beta'^2+k^2)^3} \right]^2 \qquad (3.61)$$

Bethe and Peierls (35) have evaluated the $\sigma(W)$ for the photo-disintegration of the deuteron using the zero range wave function for the ground state of the deuteron and obtain

$$\sigma_{B.P} = 8\pi/3 \left[\frac{e^2}{\hbar c} \; \frac{\hbar^2}{M} \; \frac{\epsilon^{1/2} E^{3/2}}{(E+\epsilon)^3} \right] \qquad (3.62)$$

$$\frac{\sigma_{R.C.}}{\sigma_{B.P.}} = \frac{1}{(1-\gamma r_o)} \left[1 - \left(\frac{\gamma^2+k^2}{\beta'^2+k^2}\right)^2 - \frac{4\beta'(\beta'-\gamma)(\gamma^2+k^2)^2}{(\beta'^2+k^2)^3} \right]^2 \qquad (3.63)$$

Levinger (49), Schiff (50), and Marshall and Guth (50) have calculated $\sigma(W)$ for the photodisintegration of the deuteron using Hulthén wave function and obtain

$$\sigma_{Hul} = \frac{8\pi}{3} \cdot \frac{e^2}{\hbar c} \cdot \frac{\hbar^2}{M} \frac{\epsilon^{1/2} E^{3/2}}{(E + \epsilon)^3} \frac{1}{(1 - \gamma r_o)} \left[1 - \left(\frac{\gamma^2 + k^2}{\beta^2 + k^2}\right)^2 \right]^2 \tag{3.64}$$

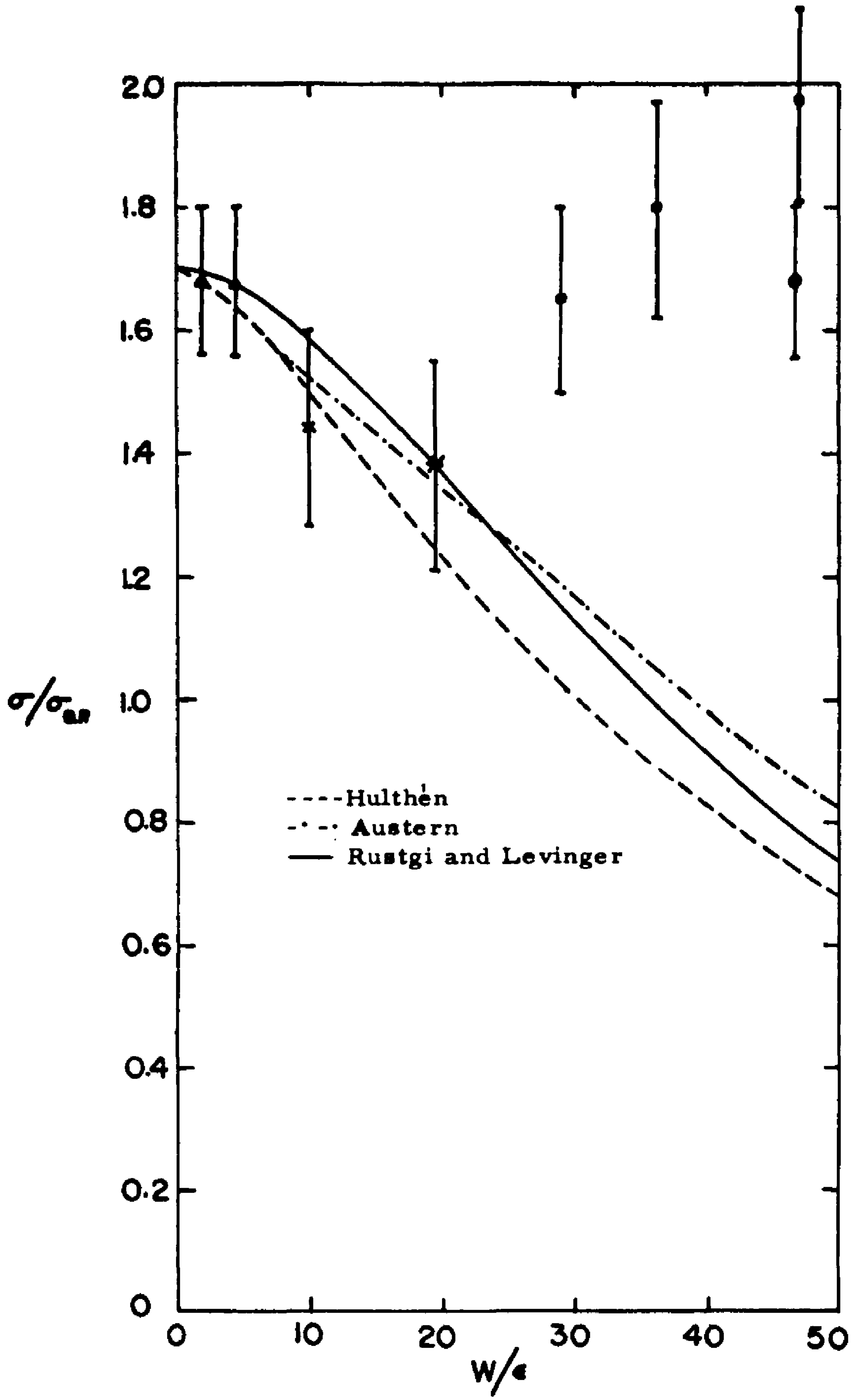
Therefore

$$\frac{\sigma_{R.C.}}{\sigma_{Hul}} = \frac{\left[1 - \left(\frac{\gamma^2 + k^2}{\beta^2 + k^2}\right)^2 - \frac{4\beta'(\beta' - \gamma)(\gamma^2 + k^2)^2}{(\beta'^2 + k^2)^3}\right]^2}{\left[1 - \left(\frac{\gamma^2 + k^2}{\beta^2 + k^2}\right)^2\right]^2} \tag{3.65}$$

and

$$\frac{\sigma_{Hul}}{\sigma_{B.P.}} = \frac{1}{(1 - \gamma r_o)} \left[1 - \left(\frac{\gamma^2 + k^2}{\beta^2 + k^2}\right)^2 \right]^2 \tag{3.66}$$

Fig. 2 shows three different calculated cross sections as well as experimental data all given relative to the Bethe-Peierls cross section. The lowest curve is the Hulthèn cross section. We see that the r^{-2} repulsive core gives rather similar results to Austern's (52) results for an infinite repulsive core of radius 0.4 f. All three calculated curves agree well with experiments up to 20 times the threshold energy ϵ ; but experiments at energies from 30 ϵ to 50 ϵ give cross sections much larger than the calculated values. (The discrepancy between calculations and experiment is reduced, but is still significant, if we use Austern's values for a core radius of 0.8 f. For example at the photon energy of 40 ϵ ; the three curves shown give $\frac{\sigma}{\sigma_{B.P.}}$ about 0.9, a core radius of 0.8 f gives $\frac{\sigma}{\sigma_{B.P.}}$ = 1.3; while the experimental data gives $\frac{\sigma}{\sigma_{B.P.}}$ about 1.85.) We conclude that present experiments up to energies of 20ϵ can not verify the changes of order 10 percent in the cross section produced by a repulsive core. The serious disagreement between calculations



2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0
10
20
30
40
50
W/ε
σ/σ
----Hulthén
-·-· Austern
—— Rustgi and Levinger

and experiment for photon energies greater than 30ϵ shows that other effects not considered here (interaction in the final state and/or mesonic effects) are of more importance than the inclusion of a repulsive core in a central potential of Serber character.

CHAPTER IV

APPLICATIONS TO H^3 AND He^3

In recent years, several authors (Gerjuoy and Schwinger (1942), Feshbach and Rarita (49), Clapp (49), Hu and Hsu (1951), Pease and Feshbach (51) and Irving (51)) have made various calculations for the ground state wave function of the three body system using a two body central and central plus tensor interactions. All these calculations have yielded reasonable values for the binding energies of H^3 and He^3 though the wave functions used were of different forms.

Verde (1950) has treated the problem of the photodisintegration of the three particle system with a Gaussian potential and a Gaussian wave function in an elementary manner. This wave function does not have an accurate asymptotic behaviour at large distances. An accurate calculation of the binding energy and photodisintegration of the three particle system has been recently carried out by Irving (51) and Gunn and Irving (51) using spin-dependent Yukawa central forces and wave function having correct asymptotic behaviour.

We shall in the present chapter calculate the electric dipole bremsstrahlung weighted cross section σ_b and σ_{int} for the H^3 and He^3 nuclei. Unfortunately there is very little experimental information available about the photodisintegration of these nuclei and therefore we shall confine ourselves only to central forces and make no attempt to compare our results with experiments. In the text, we shall use Irving's (51) wave function and spin dependent Yukawa potential for an $^1S'$ state:

$$V(r_{ij}) = - V_o \frac{e^{-Kr_{ij}}}{Kr_{ij}} \left[1 - (y + z) + (y + z) P^B_{ij} \right] \tag{4.1}$$

where r_{ij} is the relative distance between two particles i and j; and V_o and K are taken to fit the two body triplet potential.

$$V_o = 67.3 \text{ Mev. and } \frac{1}{K} = 1.17 \text{ f} \tag{4.2}$$

(See Irving (51)). y + z is the fraction of Heisenberg plus Bartlett exchange with operator P^B (P^M gives unity for an S state. The ratio (singlet well depth) / (triplet well depth)=1 - 2 (y + z) is called q by Irving, and has the numerical value 0.69 chosen to fit the two-body system.

In his calculations, Irving (51) has used a wave function of the form

$$\psi' = A\, e^{-\alpha (r_{12}^2 + r_{23}^2 + r_{31}^2)^{1/2}} \tag{4.3}$$

where 'A' is the normalisation coefficient; α is a parameter ($\alpha = 0.92 \text{ f}^{-1}$ See Irving(51)) r_{12}, r_{13} and r_{23} are the distances between the particles 1 and 2, 1 and 3, 2 and 3 respectively. (This is a special case (n = o) of Irving's general treatment). The above wave function is of course a function of the relative coordinates of the particles and does not depend upon their spin co-ordinates. It may be simplified by the following transformation.

$$\vec{\rho} = \frac{(\vec{r}_2 - \vec{r}_1) + (\vec{r}_3 - \vec{r}_1)}{2} \quad ; \vec{r} = \sqrt{\frac{3}{2}}\, (\vec{r}_2 - \vec{r}_1)$$

$$\vec{R} = \frac{\vec{r}_1 + \vec{r}_2 + \vec{r}_3}{3} \tag{4.4}$$

$\vec{r}_1$ being the position vector of the proton and $\vec{r}_2$ and $\vec{r}_3$ being the position vectors for the neutrons in a H^3 nucleus. On the application of this transformation, the wave function assumes the form

$$\psi = N\, e^{-\sqrt{2}\,\alpha(\rho^2 + r^2)^{1/2}} \tag{4.5}$$

N being the normalisation constant in the new coordinate system and $r = |\vec{r}|$ and $\rho = |\vec{\rho}|$. The normalisation constant N for the wave function ψ is easily determined.

$$\int \psi^2 d^3r\, d^3\rho = N^2 (4\pi)^2 \int_0^\infty e^{-2\sqrt{2}\,\alpha(\rho^2 + r^2)^{1/2}}\, r^2 \rho^2\, dr\, d\rho = 1 \tag{4.6}$$

using the transformation $r = R\,\mathrm{Cos}\,\theta$; $\rho = R\,\mathrm{Sin}\,\theta$, it is found that, in agreement with Irving's more general result,

$$N^2 = (\alpha^6\, 2^9 / \pi^3\, 5!) \tag{4.7}$$

We shall now calculate the E-1 bremsstrahlung weighted cross section σ_b for H^3. From Eqn. (2.54) we know that

$$\sigma_b = \frac{4\pi^2}{3} \left(\frac{e^2}{\hbar c}\right) \left\langle (\vec{r}_1 - \vec{R})^2 \right\rangle_{00} \tag{4.8}$$

But from Eqn. (4.4)

$$\vec{r}_1 - \vec{R} = -\frac{2\vec{\rho}}{3}$$

Therefore

$$(\vec{r}_1 - \vec{R})^2 = \frac{4\rho^2}{9} \tag{4.9}$$

Now

$$\left\langle (\vec{r}_1 - \vec{R})^2 \right\rangle_{00} = N^2 (4\pi)^2 \int_0^\infty e^{-2\sqrt{2}\,\alpha(\rho^2 + r^2)^{1/2}} (4\rho^2/9)\, r^2 \rho^2\, dr\, d\rho$$

$$= 7/6\alpha^2 \tag{4.10}$$

Substituting this in Eqn. (4.8), we get

$$\sigma_b = \frac{14\pi^2}{9}\left(\frac{e^2}{\hbar c}\right)\frac{1}{\alpha^2} = 1.32 \text{ mb.} \tag{4.11}$$

where $\alpha = 0.92\ f^{-1}$ from Table I in Irving (51).

From symmetry consideration, we can say that this will also be the bremsstrahlung weighted cross section for He^3. (We have also shown by explicit calculation that σ_b (H^3, = σ_b (He^3).). An explanation of the small value of σ_b for H^3 compared to the deuteron lies in small rms radius of H^3 (1.17 f) as compared to the large size of the deuteron (1.96 f).

As a next step, we shall now calculate σ_{int} for H^3. From Eqn. (2.43) we know that for central forces

$$\sigma_{int} = \int \sigma\, dW = \frac{4\pi^2 e^2 \hbar}{3Mc}\left\{1 - \frac{M(x + y/2)}{2\hbar^2}\left[\sum_i \sum_j V(r_{ij}) r_{ij}^2 P_{ij}^M\right]_{oo}\right\} \tag{4.12}$$

where we have used $\langle P_{ij}^B \rangle = 1/2$ and $\frac{y}{2}$ term comes from the Heisenberg
spin average
exchange operator. Let us now evaluate $\left[\sum_i \sum_j V(r_{ij}) r_{ij}^2\right]_{oo}$ using

$$V(r_{ij}) = -V_o \frac{(1+q)}{2} \cdot \frac{e^{-Kr_{ij}}}{Kr_{ij}}$$

$$\int \psi_o^* \left(\sum_i \sum_j r_{ij}^2\, V(r_{ij})\right)\psi_o\, d\tau = -2V_o \frac{(1+q)}{2K}\int \psi_o^*\, e^{-Kr_{12}}\, r_{12}\, \psi_o\, d\tau \tag{4.13}$$

The factor 2 comes because there are two n-p pairs.

From symmetry considerations, r_{12} may be replaced by $r_{23} = \frac{2r}{\sqrt{3}}$

$$\left[\sum_i \sum_j V(r_{ij}) r_{ij}^2\right]_{oo} = -V_o \frac{(1+q)(4\pi)^2 N^2}{K}\int_0^\infty\!\!\int_0^\infty e^{-2\sqrt{2}\alpha(\rho^2 + r^2)^{1/2}}\, \frac{2r}{\sqrt{3}}\, r^2 \rho^2\, dr\, d\rho \tag{4.14}$$

Using the transformation ρ = R Cosθ ; r = R Sinθ and carrying out the integration over R, we get

$$\left[\sum_i \sum_j V(r_{ij}) r_{ij}^2\right]_{oo} = -\frac{2}{\sqrt{3}} \frac{V_o(1+q)N^2(4\pi)^2(6!)}{K} \int_0^{\pi/2} \frac{\sin^3\theta \cos^2\theta \, d\theta}{\left(2\sqrt{2}\,\alpha + \frac{2K}{\sqrt{3}} \sin\theta\right)^7}$$

$$= -\frac{2^4\sqrt{6}\, V_o(1+q)}{\pi K \alpha} \int_0^1 \frac{(1-u^2)^{1/2} u^3 du}{(1+cu)^7} \tag{4.15}$$

where u = Sin θ and $c = \frac{K}{\sqrt{6}\alpha}$ = 0.38 (Irving (51) Table I). The value of the integral is found to be

$$\int_0^1 \frac{(1-u^2)^{1/2} u^3 du}{(1+cu)^7} = \frac{1}{720}\left\{ \frac{96 + 741c^2 + 120c^4 - 12c^6}{(1-c^2)^5} - \frac{315c\,(1+2c^2)\cos^{-1}c}{(1-c^2)^{11/2}} \right\}$$

$$= 2.58 \times 10^{-2} \tag{4.16}$$

Therefore

$$\frac{M}{2\hbar^2}\left[\sum_i \sum_j V(r_{ij}) r_{ij}^2\right]_{oo} = -\frac{M}{2\hbar^2} \frac{2^4\sqrt{6}\, V_o(1+q)}{\pi K \alpha} (2.58 \cdot 10^{-2})$$

$$= -0.55 \tag{4.17}$$

Combining all the results we get,

$$\sigma_{int} = \frac{4}{3} \frac{\pi^2 e^2 \hbar}{Mc}\left[1 + 0.55\left(x + \frac{y}{2}\right)\right]$$

$$= 40\left[1 + 0.55\left(x + \frac{y}{2}\right)\right] \text{ Mev.-mb} \tag{4.18}$$

Due to the lack of experimental data, we shall not attempt any comparison with experiments for H^3 or He^3. We might remark at this place that due to the assumption of charge independence the results are true both for H^3 and He^3

CHAPTER V

APPLICATIONS TO He^4

Since a detailed study of the deuteron and two nucleon scattering has not yielded a unique nuclear interaction, in recent years several physicists have investigated nuclear problems involving more than two nucleons so that adequacy or inadequacy of the two body interaction for such problems may be demonstrated. Several attempts to calculate the binding energy of three and four particle nuclei using a mixture of central and tensor forces have been made by Gerjuoy and Schwinger (42), Feshbach and Rarita (49), Clapp (49), Hu and Hsu (1951) and Pease and Feshbach (51). Pease and Feshbach (51) using a phenomenological nuclear interaction potential due to Feshbach and Schwinger (51) and consisting of central and tensor forces of the form

$$V(r) = -V_o\left[\left\{1 - \frac{g}{2} + \frac{g}{2}\left(\vec{\sigma}_1 \cdot \vec{\sigma}_2\right)\right\} J(r) + \gamma' S_{12} K(r)\right] \tag{5.1}$$

where
$$J(r) = \frac{e^{-r/r_c}}{r/r_c} \quad \text{and} \quad K(r) = \frac{e^{-r/r_t}}{r/r_t} \tag{5.2}$$

have demonstrated that such an interaction yields a reasonable value for the binding energy of the triton for the following choice of the parameters V_o, γ', g, r_c and r_t.

$$r_c = 1.184 \text{ f}; \; r_t = 1.67 \text{ f}; \; V_o = 46.1 \text{ Mev.}$$
$$\gamma' = 0.54, \quad g = -0.004 \tag{5.3}$$

In recent years, Irving (51, 52) has made two calculations to evaluate the binding energy of He^4, one using purely central interaction of the Yukawa and exponential types and the other using Feshbach and Schwinger interaction potential given in (5.1).

In order to make a comparison between central and central and tensor force calculations, we shall confine ourselves to Yukawa type potential, which for the central force case may be written as

$$V_c(r_{ij}) = - V_o \frac{e^{-r_{ij}/r_c'}}{r_{ij}/r_c'} \tag{5.4}$$

To calculate the binding energy of He^4, Irving has assumed an exponential type of wave function of the form.

$$\phi_c = A^{1/2} \exp \left\{ - \alpha_c (r_{12}^2 + r_{13}^2 + r_{14}^2 + r_{23}^2 + r_{24}^2 + r_{34}^2)^{1/2} \right\} \tag{5.5}$$

where 1, 2 denote the neutron and 3, 4 the proton co-ordinates. (We are here considering a special case of Irving's (51) general case namely n = o); r_{12}, etc. are the distances between the particles 1 and 2, etc. To simplify (5.5) Irving has used the following transformation:

$$\vec{u} = \frac{\vec{r_3} + \vec{r_4} - \vec{r_2} - \vec{r_1}}{2} \quad ; \quad \vec{v} = \frac{\vec{r_2} - \vec{r_1}}{\sqrt{2}}$$

$$\vec{w} = \frac{\vec{r_4} - \vec{r_3}}{\sqrt{2}} \quad ; \quad \vec{R} = \frac{\vec{r_1} + \vec{r_2} + \vec{r_3} + \vec{r_4}}{4} \tag{5.6}$$

where $\vec{r_1}$, $\vec{r_2}$, $\vec{r_3}$ and $\vec{r_4}$ are the position vectors of the particles 1, 2, 3 and 4, respectively. This transformation when applied to (5.5) gives

$$\psi_c = N_c^{1/2} \exp \left\{ -2 \alpha_c (u^2 + v^2 + w^2)^{1/2} \right\} \tag{5.7}$$

$N_c^{1/2}$ being the normalisation constant in the new co-ordinate system and can be easily determined.

$$\int \psi_c^2 \, d\vec{u} \, d\vec{v} \, d\vec{w} = N_c (4\pi)^3 \int_0^\infty e^{-4\alpha_c (u^2+v^2+w^2)^{1/2}} u^2 v^2 w^2 du\, dv\, dw = 1 \tag{5.8}$$

Using the transformation

$$u = R \sin\theta \ \cos\varphi \ .$$

$$v = R \sin\theta \ \sin\varphi \ .$$

$$w = R \cos\theta \ . \tag{5.9}$$

and carrying out the integration over R, θ and φ, it is found that

$$N_c = \frac{2^6 \alpha_c^9}{3 \pi 4} \tag{5.10}$$

In his first calculation, Irving (51) found that a radial wave function of an exponential type yields a large excess binding energy in the purely central force case. Since an exponential type wave function has a correct asymptotic behaviour, in his second paper, Irving (52) has taken tensor forces into consideration and has assumed the ground state wave function to be a mixture of the principal ^1S_o and the principal $^5D_o^{(1)}$ state. (An analysis of the various wave functions of the states that can occur in the ground state of He^4 is given by Gerjuoy and Schwinger (41).). In the absence of the spin orbit coupling due to the tensor force, the ground state of He^4 is ^1S_o. The introduction of a tensor interaction gives rise to several D states, but Irving has considered one which has no radial nodes and may be taken to be the principal $^5D_o^{(1)}$ state. Its angular and spin part has the form (see Irving (52).).

$$^5D_o^{(1)} = \left\{ \sum_{i>j=1}^{4} D(r_{ij}) \right\} \chi \tag{5.11}$$

where

$$D(r_{ij}) = r_{ij}^2 \ S_{ij} \ ;$$

$$S_{ij} = \frac{3\,(\vec{\sigma}_i \cdot \vec{r}_{ij})\,(\vec{\sigma}_j \cdot \vec{r}_{ij})}{r_{ij}^2} - (\vec{\sigma}_i \cdot \vec{\sigma}_j)$$

and $\chi = 1/2\,(\chi_1^+\chi_2^- - \chi_1^-\chi_2^+)(\chi_3^+\chi_4^- - \chi_3^-\chi_4^+)$ is the spin wave function. Using the fact that the tensor operator vanishes when it acts on a singlet spin state and applying the transformation (5. 6), (5. 11) assumes the more compact form.

$$6(\vec{\sigma}_1 . \vec{v})(\vec{\sigma}_3 . \vec{w}) + 6(\vec{\sigma}_1 . \vec{w})(\vec{\sigma}_3 . \vec{v}) - 4(\vec{v} . \vec{w})(\vec{\sigma}_1 . \vec{\sigma}_3) \tag{5. 12}$$

For the radial wave function in the spatially symmetric case, Irving (52) has used an expression similar to (5. 5). The complete 'D' state wave function is obtained on multiplying the angular spin part (5. 12) with a spatial radial function. The complete wave function representing a mixture of the ^1S_o and $^5D_o^{(1)}$ states may be written in the form.

$$\Psi = \frac{1}{(1+C^2)^{1/2}}\Big\{ N_S^{1/2} e^{-2\alpha(u^2+v^2+w^2)} + C\Big[6(\vec{\sigma}_1 . \vec{u})(\vec{\sigma}_3 . \vec{w}) + 6(\vec{\sigma}_1 . \vec{w})(\vec{\sigma}_3 . \vec{v}) - 4(\vec{v} . \vec{w})(\vec{\sigma}_1 . \vec{\sigma}_3)\Big] N_D^{1/2} e^{-2\beta(u^2+v^2+w^2)^{1/2}}\Big\} \tag{5. 13}$$

where C^2 determines the amount of 'D' state in the mixture, assuming S and D wave functions to be normalised to unity separately. The normalisation constant $N_S^{1/2}$ is evaluated as in the central force case (See Eqn. (5. 10).).

$$N_S = \frac{2^6 \alpha^9}{3 \pi^4} \tag{5. 14}$$

and N_D is determined as follows.

$$N_D \int_o^\infty e^{-4\beta(u^2+v^2+w^2)^{1/2}} d\vec{u}\, d\vec{v}\, d\vec{w} \left\langle {}^5D_o^{(1)*} \,\middle|\, {}^5D_o^{(1)} \right\rangle = 1. \tag{5. 15}$$

From the appendix, the spin matrix element

$$\left\langle {}^5D_o^{*(1)} \middle| {}^5D_o^{(1)} \right\rangle = 72\, v^2 w^2 + 24\, (\vec{v}\cdot\vec{w})^2 \tag{5.16}$$

Therefore

$$N_D \int_0^\infty e^{-4\beta\,(u^2+v^2+w^2)^{1/2}} \left\{ 72\, v^2 w^2 + 24 (\vec{v}\cdot\vec{w})^2 \right\} d\vec{u}\, d\vec{v}\, d\vec{w} = 1$$

Applying the transformation (5.9) and carrying out the integrations we get

$$N_D = \frac{2^7\, \beta^{13}}{5^2 3^4 \pi^4} \tag{5.17}$$

Using Pease-Feshbach (51) parameters (5.3) (also see Ch. II). Irving (52) has made a variation calculation on the binding energy of the alpha particle. Irving has been able to fit the binding energy of the alpha particle reasonably well with $\alpha = 1.194\ f^{-1}$ and $\beta = 1.792\ f^{-1}$. With these parameters, the binding energy of He^4 turns out to be 24.2 Mev. which is fairly close to the experimental value. The inclusion of a second 'D' state (Clark 53) leads to a fairly large increase in binding energy (29.87 Mev.) of the He^4 nucleus, compared to the value (20.85 Mev.) when only one 'D' state is included, using different trial wave functions than those used by Irving.

We shall first in the following make a comparison of the rms radius calculated from the wave functions (5.7) and (5.13) with the size of the alpha particle measured by Blankenbecler and Hofstadter (56). Blankenbecler and Hofstadter have made measurements on the scattering of electrons from the alpha particle in He^4 gas at 400 Mev. and a form factor analysis of their data gives an rms radius of 1.61 fermis for the He^4 nucleus. In order to make a comparison, we shall have to calculate

$\langle(\vec{r}_3-\vec{R})^2\rangle_{oo}$ or $\langle(\vec{r}_4-\vec{R})^2\rangle_{oo}$ where $\vec{r}_3$ and $\vec{r}_4$ are the proton co-ordinates in the He^4 nucleus.

From Eqn. (5.6), it is evident that

$$\vec{r}_3 - \vec{R} = \frac{\vec{u}}{2} - \frac{\vec{w}}{\sqrt{2}}$$

$$\vec{r}_4 - \vec{R} = \frac{\vec{u}}{2} - \frac{\vec{w}}{\sqrt{2}} \qquad (5.18)$$

Since $\langle(\vec{r}_3 - \vec{R})^2\rangle_{oo} = \langle(\vec{r}_4 - \vec{R})^2\rangle_{oo}$ we conclude that the expectation value of $\langle(\vec{w}\cdot\vec{u})\rangle_{oo}$ should be zero and we have to evaluate $\langle\frac{u^2}{4} + \frac{w^2}{2}\rangle_{oo}$

For the central force case (5.7)

$$\langle(u^2/4 + w^2/2)\rangle^c_{oo} = \frac{45}{32\alpha_c^2} = 0.616 \times 10^{-26}\ cm^2.$$

where we have used $\alpha_c = 1.511\ f^{-1}$

Therefore

$$\sqrt{\langle(\vec{r}_3 - \vec{R})^2\rangle^c_{oo}} = 0.78\ f \qquad (5.19)$$

For the tensor force case (5.13)

$$\langle\frac{u^2}{4} + \frac{w^2}{2}\rangle^T_{oo} = \frac{1}{(1+C^2)}\left\{\int \psi_S\,(u^2/4 + w^2/2)\,\psi_S\, d\vec{u}\, d\vec{v}\, d\vec{w} + C^2\int \psi_D\,(u^2/4 + w^2/2)\,\psi_D\, d\vec{u}\, d\vec{v}\, d\vec{w}\right\}$$

$$= \frac{1}{(1+C^2)}\left\{\left(\frac{15}{32\alpha^2} + \frac{15}{16\alpha^2}\right) + C^2\left(\frac{21}{32\beta^2} + \frac{35}{16\beta^2}\right)\right\} \qquad (5.20)$$

Using $\alpha = 1.94\ f^{-1}$, $\beta = 1.792\ f^{-1}$ and $C = -.162$

$$\langle\frac{u^2}{4} + \frac{w^2}{2}\rangle_{oo} = \frac{1}{1.0262}\ (.9855 + .0232)\ f^2 = 0.9829\ f^2 \qquad (5.21)$$

Therefore

$$\sqrt{\langle(\vec{r}_3 - \vec{R})^2\rangle_{oo}} = .99\ f \doteqdot 1.0\ f$$

which is appreciably larger than (5. 19) but about 2/3 of the experimental value. Dalitz and Ravenhall (Hofstadter 56) have recently computed an rms radius from the wave function of Clark (53) who also used a variational method to fit the binding energy of the alpha particle. The resulting radius is found to be 2/3 of the required size. Since Clark (53) has included two D states in his wave function while Irving has used only one, we conclude that the additional D states have little effect on the rms radius of the alpha particle (Also note in Eqn. (5. 2) that the principal D state contributes only 2-1/2 per cent to the mean square radius). There are several possibilities for this serious disagreement between theory and experiment.

1) Irving and Clark both have not included the finite size of the proton (0. 77 f). An inclusion of this finite size of the proton, but treatment of the neutron as a point-charge increases the rms radius of the α particle to about 1. 30 f; which is still 0. 30 f below the experimental value. (This procedure gives agreement with McIntyre's (56) measurements of electron-deuteron scattering; but it violates the assumption of charge symmetry in nuclear physics).

2) Recently Yamada et al (56) have calculated the effect of hard core on the binding energy of H^3 and He^3 and they find that the hard core reduces the binding energy of H^3 and with properly chosen parameters decreases the coulomb energy of He^3 by 25 per cent to give the experimental value. The rms radius of the 3-body system is increased by roughly 25 per cent. We therefore believe that in addition to all the D

states, an inclusion of hard core might possibly lead to the right binding energy and rms radius of the α particle.

3) A third alternative will be, as suggested by McIntyre (56) to assume that the charge density found from scattering experiments at high energies (400 Mev.) cannot be directly related to the $|\psi|^2$ for solution of the nuclear Schrödinger equation. (Yennie, Lévy, Ravenhall, unpublished).

As a next step, we shall now calculate the E-I bremsstrahlung weighted cross section σ_b first with central (5.5) and later on with tensor wave functions (5.13).

Modifying Eqn. (2.54) slightly, we find that

$$\sigma_b^c = \frac{4\pi^2}{3}\left(\frac{e^2}{\hbar c}\right)\left\langle\left(\sum_{i=3,4}(\vec{r}_i-\vec{R})\right)^2\right\rangle_{oo} \tag{5.22}$$

But from (5.18)

$$(\vec{r}_3-\vec{R})+(\vec{r}_4-\vec{R})=\vec{u}$$

Therefore for the central force case

$$\begin{aligned}\sigma_b^c &= \frac{4\ \ ^2}{3}(e^2/\hbar c)\langle u^2\rangle_{oo}^c\\ &= \frac{4\pi^2}{3}(e^2/\hbar c)\,(15/8\,\alpha_c^2).\\ &= 0.8\text{ mb.}\end{aligned} \tag{5.23}$$

For the tensor case

$$\begin{aligned}\langle u^2\rangle_{oo}^T &= \frac{1}{(1+C^2)}\left[\int\psi_S^* u^2\psi_S\,d\vec{u}d\vec{v}d\vec{w} + C^2\int\psi_D^* u^2\psi_D\,d\vec{u}\,d\vec{v}\,d\vec{w}\right]\\ &= \frac{1}{(1+C^2)}\left[\frac{15}{8\alpha^2}+\frac{21C^2}{8\beta^2}\right]\end{aligned} \tag{5.24}$$

(See Eqn. 5.20)

Therefore

$$\sigma_b^{\,T} = \frac{4\pi^2}{3}\left(\frac{e^2}{\hbar c}\right)\frac{1}{(1+C^2)}\left(\frac{15}{8\alpha^2} + \frac{21\,C^2}{8\,\beta^2}\right) \tag{5.25}$$

Substituting the values of α, C and β, we get

$$\sigma_b^{\,T} = 1.23 \text{ mb.} \tag{5.26}$$

which indicates that the tensor force tends to increase the E-1 bremsstrahlung weighted cross section. Since the spin of alpha particle is zero, therefore there are no magnetic dipole transitions.

To evaluate σ_{int}, we shall go back to Eqn. (2.53) and apply it to the alpha particle wave function. Since for an alpha particle

$$\langle P^M \rangle_{oo} \longrightarrow 1 \quad \text{and} \quad \langle P^H \rangle_{oo} \longrightarrow 1/2$$

We get

$$\int \sigma \, dW = \frac{2\pi^2 e^2 \hbar}{Mc}\left\{ 1 - \frac{M(x+y/2)}{3\hbar^2}\left[\sum_i \sum_j J(r_{ij}) r_{ij}^2 V_o\right]_{oo} - \right.$$

$$\left. - \frac{\gamma' M(x'+y'/2)}{3\hbar^2}\left[\sum_{ij}\sum S_{ij} K(r_{ij}) r_{ij}^2 V_o\right]_{oo}\right\} \tag{5.27}$$

which for the central force reduces to ($x' = o$; $y' = o$).

$$\int \sigma^c \, dW = \frac{2\pi^2 e^2 \hbar}{Mc}\left\{ 1 - \frac{M(x+y/2)}{3\hbar^2}\left[\sum_i \sum_j V_c(r_{ij}) r_{ij}^2\right]_{oo}\right\} \tag{5.28}$$

Now

$$\left\langle \sum_{ij}\sum V_c(r_{ij}) r_{ij}^2 \right\rangle_{oo} = \int \psi_o^* \left(\sum_i \sum_j V_c(r_{ij}) r_{ij}^2\right)\psi_o \, d\vec{u}\, d\vec{v}\, d\vec{w}$$

Using (5.4), and (5.7) we get

$$\left\langle \sum_i \sum_j V_c(r_{ij}) r_{ij}^2 \right\rangle_{oo} = -4\, V_o N_c \int \frac{e^{-4\alpha_c (u^2+v^2+w^2)^{1/2} - r_{13}/r_c'}}{(r_{13}/r_c')} r_{13}^2 \, d\vec{u}\, d\vec{v}\, d\vec{w} \tag{5.29}$$

Since the value of (5.29) is unaltered by interchanging r_1 and r_4, the integral becomes

$$\left\langle \sum_i \sum_j V_c(r_{ij}) r_{ij}^2 \right\rangle_{oo} = -4 V_o N_c \int \frac{e^{-4\alpha_c (u^2+v^2+w^2)^{1/2} - r_{34}/r_c'}}{(r_{34}/r_c')} r_{34}^2 \, d\vec{u}\, d\vec{v}\, d\vec{w} \tag{5.30}$$

Using the transformations (5.6) and (5.9) and carrying out the integrations over R and φ, we get

$$\left\langle \sum_i \sum_j V_c(r_{ij}) r_{ij}^2 \right\rangle_{oo} = - \frac{2^5 V_o \pi^4 N_c}{(\sqrt{2}/r_c')^{11}} \; 9\,!\, A(a') \tag{5.31}$$

where $\quad A(a') = \int_o^1 \frac{(1-u^2)^2 \, u^3 \, du}{(a' + u)^{10}}$

$$a' = (2\sqrt{2}\; \alpha_c \; r_c') \; ; \; u = \cos\theta \tag{5.32}$$

and the value of A(a') is given in the appendix B

$$\sigma_{int}^{c} = \frac{2\pi^2 e^2 \hbar}{Mc} \left[1 + \frac{M(x+Y/2)}{3\hbar^2} \frac{2^5 V_o \pi^4 N_c}{2/r_c')^{11}} \; 9 \, ! \; A(a') \right]$$

Evaluating this numerically, we obtain

$$\sigma_{int}^{c} = 60 \left[1 + 0.97(x + Y/2) \right] \tag{5.33}$$

We shall now calculate σ_{int}^{T} with tensor forces and wave functions

$$\left\langle \sum_i \sum_j V(r_{ij}) r_{ij}^2 \right\rangle_{oo} = \frac{1}{(1 + C^2)} \int (\psi_S^* + C \psi_D^*) \left(\sum_i \sum_j r_{ij}^2 (V_c(r_{ij}) + V_t(r_{ij})) \right) (\psi_S + C \psi_D) \, d\tau \tag{5.34}$$

$$= \frac{1}{(1+C^2)} \Big[x \int \psi_S^* \Big(\sum_i \sum_j (V_c(r_{ij}) r_{ij}^2 \Big) \psi_S \, d\tau + 2Cx' \int \psi_S^* \Big(\sum_i \sum_j (V_t(r_{ij}) r_{ij}^2 \Big) \psi_D \, d\tau$$
$$+ C^2 x \int \psi_D^* \Big(\sum_i \sum_j (V_c(r_{ij}) r_{ij}^2 \Big) \psi_D \, d\tau + C^2 x' \int \psi_D^* \Big(\sum_i \sum_j (V_t(r_{ij}) r_{ij}^2 \Big) \psi_D \, d\tau \Big] \tag{5.35}$$

The evaluation of the integrals is fairly straight forward. We shall evaluate them in three cases here

$$\text{(i)}\quad 2\,C\,x' \int \psi_S^* \left(\sum_i \sum_j (V_t(r_{ij}) r_{ij}^2\right) \psi_D \, d\tau \text{ where } V_t = -\gamma V_o S_{ij} \frac{e^{-r_{ij}/r_t}}{r_{ij}/r_t} \tag{5.36}$$

The expression may be written as

$$-8\,\gamma' C\, x'\, V_o \int \phi \phi_S^* \frac{e^{-r_{13}/r_t}}{(r_{13}/r_t)} r_{13}^2 \langle {}^1S_o^* | S_{13} | {}^5D_o^{(1)} \rangle \, d\tau \tag{5.37}$$

But from the appendix 'A', the spin matrix element

$$\langle {}^1S_o^* | S_{13} | {}^5D_o^{(1)} \rangle = \frac{18\,(\vec{r}_{13}\cdot\vec{\rho}_1)(\vec{r}_{13}\cdot\vec{\rho}_2)}{r_{13}^2} - 6\,(\vec{\rho}_1 \cdot \vec{\rho}_2) \tag{5.38}$$

and since the value of the above integral is unchanged by interchanging r_1 and r_4, the above expression becomes

$$-8\,\gamma' C\, V_o N_S^{1/2} N_D^{1/2} x' \int \frac{e^{-2(\alpha+\beta)(u^2+v^2+w^2)^{1/2} - \frac{\sqrt{2}w}{r_t}}}{(\sqrt{2}/r_t)\, w} \left[18 \left\{ \frac{(\vec{w}.\vec{u})^2}{w^2} - \frac{(\vec{w}.\vec{v})^2}{2w^2} - \frac{w^2}{2} \right\} - 6 \left\{ u^2 - 1/2(v^2+w^2) \right\} \right] (2w^2)\; d\vec{u}\, d\vec{v}\, d\vec{w} \tag{5.39}$$

This expression is then evaluated by using the transformation (5.9) and we get

$$\frac{(12)!\, 2^5\, \pi^4}{(\sqrt{2}/r_t)^{13}}\; \gamma' C\, V_o\, x'\, N_S^{1/2}\, N_D^{1/2}\; D(e)$$

where D(e) denotes the integral.

$$D(e) = \int_0^1 \frac{(1-u^2)^2\, u^5\, du}{(e+u)^{12}} \tag{5.40}$$

where $e = \sqrt{2}\,(\alpha+\beta)\, r_t$ and $u = \cos\theta$ and the value of the integral is given in the appendix B. Similarly we can show that

$$x \int \psi_S^* \left(\sum_i \sum_j (V_c(r_{ij})\, r_{ij}^2\right) \psi_S \, d\tau = -\frac{V_o N_S (1-g)\, \pi^4 2^5 x (9\,!) E(a')}{(\sqrt{2}/r_c)^{11}} \tag{5.41}$$

where $E(a')$ denotes the integral $\int_0^1 \frac{(1-u^2)^2\, u^3\, du}{(a' + u)^{10}}$ given in the appendix B and $a' = 2\sqrt{2}\,\alpha r_c)$. Now considering the third term

$$C^2 x \int \psi_D^* \left(\sum_i \sum_j (V_c(r_{ij}) r_{ij}^2\right) \psi_D \, d\tau = -4\, C^2 V_o N_D x \int \frac{e^{-4\beta(u^2+v^2+w^2)^{1/2} - \frac{\sqrt{2}\, w}{r_c}}}{\left(\frac{\sqrt{2}\, w}{r_c}\right)}$$

$$\left[72\, v^2 w^2 + 24\, (\vec{v}.\vec{w})^2\right] \, d\vec{u}\, d\vec{v}\, d\vec{w} \qquad (5.42)$$

where the value of the spin matrix element $\left\langle {}^5D_o^{*(1)} \middle| {}^5D_o^{(1)} \right\rangle$ has been used from the appendix A.

Applying the transformation (5.9) and carrying out the integration over R and φ, we get

$$\frac{-20\, C^2\, V_o\, 2^6\, \pi^4\, N_D\, (13)!\quad F(a)}{(\sqrt{2}/r_c)^{15}} \qquad (5.43)$$

where $F(a) = \int_0^1 \frac{(1-u^2)^3\, u^5\, du}{(a+u)^{14}}$; $a = 2\sqrt{2}\;\beta\, r_c$

and the value of the integral is given in the appendix B.

We shall now evaluate the fourth term $C^2 x' \int \psi_D^* \left(\sum_i \sum_j (V_t(r_{ij}) r_{ij}^2\right) \psi_D \, d\tau$

Substituting the value of $V_t(r_{ij})$ (5.36), we get

$$-\gamma' x' V_o N_D C^2 \int \psi_D^* \left(\sum_i \sum_j (S_{ij} \frac{e^{-r_{ij}/r_t}}{r_{ij}/r_t}\, r_{ij}^2\right) \psi_D \, d\tau \qquad (5.44)$$

We shall evaluate this integral in two steps, first when S_{ij} is S_{12} and S_{34}. For these cases, the integral reduces to

$$-2\,\gamma' V_o x' N_D C^2 \int \frac{e^{-4\beta(u^2+v^2+w^2)^{1/2} - r_{12}/r_t}}{(r_{12}/r_t)} \left\langle {}^5D_o^{*(1)} \middle| S_{12} \middle| {}^5D_o^{(1)} \right\rangle r_{12}^2\, d\tau$$

$$(5.45)$$

But from the appendix 'A'

$$\left\langle {}^5D_o^{*(1)} \right| S_{12} \left| {}^5D_o^{(1)} \right\rangle = -24 \left\{ 3\ v^2 w^2 + 5\ (\vec{v}.\vec{w}.)^2 \right\} \qquad (5.46)$$

Inserting the value of the spin matrix element in (5.45) and applying the transformation (5.9), we get on carrying out the integration over R and φ :

$$\frac{2^3 \pi^4 (14)\ \gamma' V_o N_D C^2 x' (13)\ !\ G(a'')}{(\sqrt{2}/r_t)^{15}} \qquad (5.47)$$

where

$$G(a'') = \int_o^1 \frac{(1-u^2)^3 u^5 du}{(a'' + u)} \quad ; \quad a'' = 2\sqrt{2}\ \beta\ r_t$$

and the value of the integral is given in the appendix B.

Considering the other two terms (S_{13} and S_{24}) we can write the integral in the form

$$-2\ \gamma' V_o x' N_D C^2 \int \frac{e^{-4\beta(u^2+v^2+w^2)^{1/2} - r_{13}/r_t}}{(r_{13}/r_t)}\ r_{13}^2 \left\langle {}^5D_o^{*(1)} \right| S_{13} \left| {}^5D_o^{(1)} \right\rangle d\tau \qquad (5.48)$$

where from the appendix A, we know that the spin matrix element

$$\left\langle {}^5D_o^{*(1)} \right| S_{13} \left| {}^5D_o^{(1)} \right\rangle = \left\{ 42\,(\vec{\rho}_1.\vec{\rho}_2)^2 - 18\,\rho_1^2 \rho_2^2 - 72\,\frac{(\vec{r}_{13}.\vec{\rho}_1)(\vec{r}_{13}.\vec{\rho}_2)(\vec{\rho}_1.\vec{\rho}_2)}{r_{13}^2} + 54\,\frac{\left[(\vec{\rho}_1 \times \vec{r}_{13}).\vec{\rho}_2\right]^2}{r_{13}^2} \right\} \qquad (5.49)$$

Substituting (5.49) in (5.48), we have on interchanging r_1 and r_4

$$-2\ \gamma' V_o x' N_D C^2 \int \frac{e^{-4\beta(u^2+v^2+w^2)^{1/2} - r_{34}/r_t}}{(r_{34}/r_t)} \left\{ 42\,(\vec{\rho}_1.\vec{\rho}_2)^2 - 18\,\rho_1^2 \rho_2^2 - 72\,\frac{(\vec{r}_{34}.\vec{\rho}_1)(\vec{r}_{34}.\vec{\rho}_2)(\vec{\rho}_1.\vec{\rho}_2)}{r_{34}^2} + 54\,\frac{\left[(\vec{\rho}_1 \times \vec{r}_{34}).\vec{\rho}_2\right]^2}{r_{34}^2} \right\} d\tau$$

On applying transformation (5.9) and carrying out the integrations over R and φ we get

$$\frac{-13\,\gamma'\,N_D\,V_o\,C^2\,2^3\,\pi^4\,(13)!}{(\sqrt{2}/r_t)^{15}}\quad G(a'') \tag{5.50}$$

where $G(a'')$ is the same as in (5.47)

Combining all the results (5.40), (5.41), (5.43), (5.47), (5.50) we get

$$\sigma_{int} = \frac{2\pi^2 e^2 \hbar}{Mc}\left[\,1 + \frac{M}{3\hbar^2}\left\{\frac{(12)!\,2^5\,\pi^4\,\gamma' C\,V_o N_S^{1/2} N_D^{1/2} D(e)\,(x'+y'/2)}{(\sqrt{2}/r_t)^{13}} + \right.\right.$$

$$+\frac{(x+y/2)N_S(1-g)\,2^3\pi^4(9!)V_o E(a')}{(\sqrt{2}/r_c)^{11}} + \frac{20C^2V_o 2^6\pi^4 N_D (13)!\,F(a)(x+y/2)}{(\sqrt{2}/r_c)^{15}} +$$

$$\left.\left. - \frac{\gamma' N_D V_o C^2 2^3\,\pi^4 (13)!\,G(a'')\,(x'+y'/2)}{(\sqrt{2}/r_t)^{15}}\right\}\right] \tag{5.51}$$

Evaluating it numerically we get

$$\sigma_{int} = 60\,\Big[1 + 0.1815\,(x'+y'/2) + 0.6710\,(x+y/2) + 0.0240\,(x+y/2) + 0.0001\,(x'+y'/2)$$

$$= 60\,\Big[1 + .695\,(x+y/2) + .182\,(x'+y'/2)\Big]\,\text{Mev.-mb} \tag{5.52}$$

Levinger (54) has made a sum rule calculation of electric dipole transitions in the nuclear photoeffect using a simple harmonic oscillator independent particle model. For the particular case of He^4 he finds that $\sigma_b = 2.3$ mb.

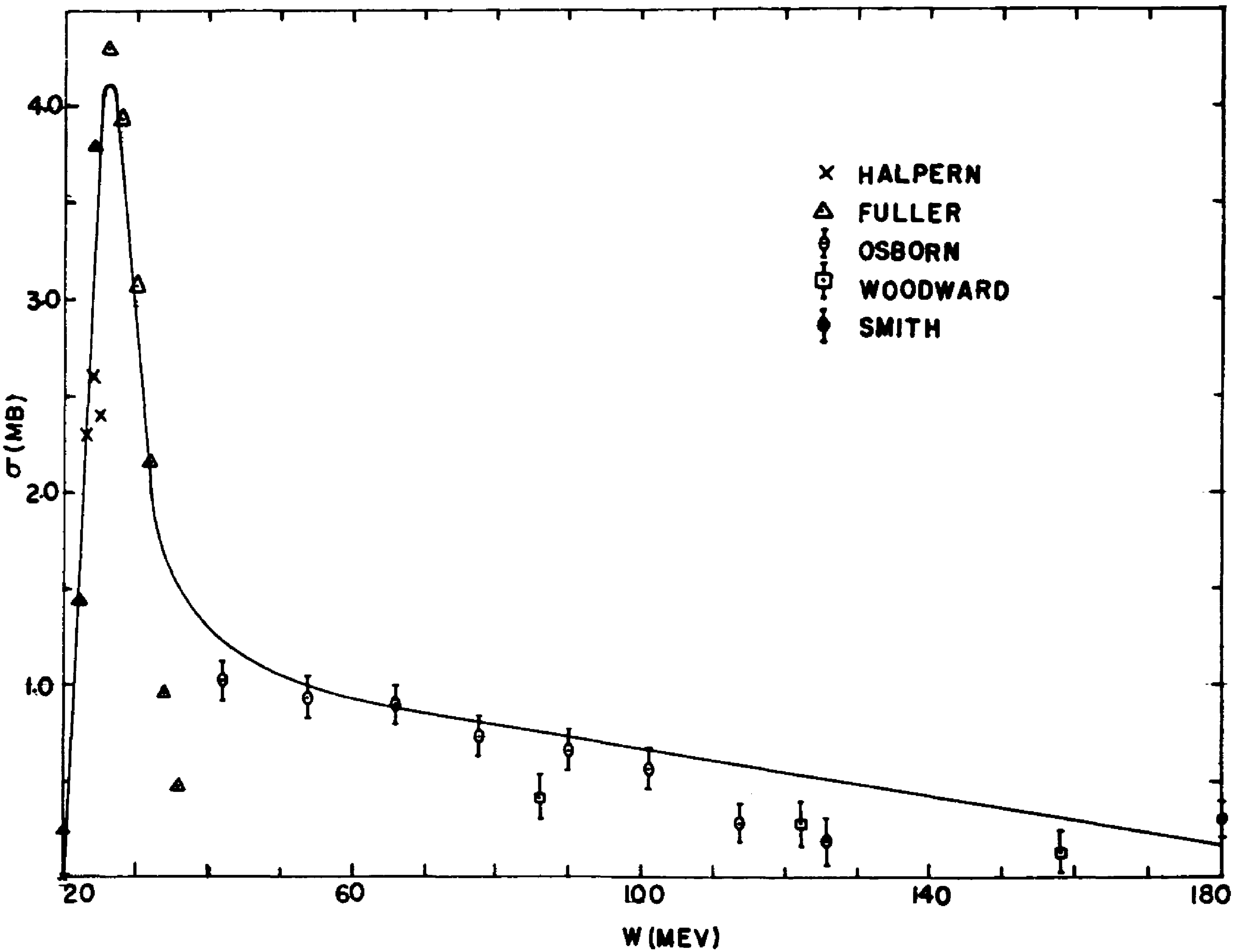
For the nuclear radius parameter $r_o = 1.2$, and quasi-Yukawa potential he finds that $\sigma_{int} = 60\,\left[1 + 0.84x\right]$ while for $r_o = 1.5$; $\sigma_{int} = 60\,\left[1 + 0.72x\right]$. A comparison with our results shows that the two results are comparable. We might remark at this point that Levinger's wave function did not have any correlations between nuclear particles, while our wave function does have some correlation.

As a last step in this chapter, we shall make a comparison with various experiments on the photodisintegration of He^4 nucleus. Experiments on the photodisintegration of alpha particle have been performed by Fuller (54), Osborne (55), Halpern et al (54), Benedict and Woodward (51) and Smith and Barton (55). Benedict and Woodward measured the cross sections for the production of protons from He^4 using the bremsstrahlung beam of the Cornell synchroton for photon energies ranging from about 45 to 120 Mev. and have expressed their results in terms of differential cross sections. We calculate the total cross section from their graph by assuming an angular distribution of the form $(a + b \sin^2\theta)$. Fuller (54) irradiated helium gas at atmospheric pressure with 26, 29, and 40 Mev. bremsstrahlung spectra and measured the energy and angular distributions of the protons produced by the photodisintegration of He^4 using nuclear emulsion technique. The results of his measurements were read from a curve for σ_{total} vs photon energy. (These σ's are increased by 20 per cent due to recalibration of the photon monitor - private communication). Both Woodward and Fuller obtain photon energies using the assumption that the dominant reaction giving protons is $He^4(\gamma, p)He^3$. Fuller has checked this assumption in his energy range by use of different betatron energies. Halpern et al (54) measured the cross section for the photodisintegration of He^4 by direct detection of the outgoing neutrons and analyzed their yield curve by the photon difference method. We have read their cross sections also from their curve of photoneutron cross section for He^4. deSaussure and Osborne have investigated the photodisintegration reaction: $\gamma + He^4 \longrightarrow He^3 + n$ by measuring the energy and angular distribution of the He^3 recoil nuclei for photon energies between 40 and 120 Mev.

Smith and Barton ((55) and private communication) have measured proton yield and neutron-proton coincidences from alpha particles irradiated by 285 Mev. bremsstrahlung. Analysis of the neutron proton coincidences for 65 Mev. protons on the quasi-deuteron model gives an alpha particle cross section of 0.2 mb at a photon energy of 180 Mev. Since some protons were not in coincidence with neutrons, Smith suggests using a total cross section about 50 per cent larger, i.e., 0.3 mb.

(In the following we shall assume that $\sigma(\gamma, n) = \sigma(\gamma, p)$ for the photodisintegration of He^4. Thus we double the proton cross sections of Woodward and of Fuller and the neutron cross sections of Halpern and of Osborne to find the total cross sections of photodisintegration of the alpha particle).

Combining all the measurements, we know the total cross section from threshold to 150 Mev., with a standard error of about 10 per cent for the better measurements. The measurements are not completely consistent (as shown in the Fig. 3). More experimental measurements are in progress (Goldwasser, private communication). We have



HALPERN
FULLER
OSBORN
WOODWARD
SMITH
σ (MB)
4.0
3.0
2.0
1.0
20
60
100
140
180
W (MEV)

calculated σ_b and σ_{int} from the preliminary curve shown in Fig. 3 and the following table shows a comparison between theory and experiments.

	σ_b	σ_{int}
Experiment	2.7 mb	124 Mev. -mb
Central	0.8 mb	$60\left[1 + 0.97\,(x + y/2)\right]$
Central and tensor	1.23 mb	$60\left[1 + 0.695\,(x + y/2) + 0.182\,(x'+y'/2)\right]$

Assuming $x = x'$, $y = y'$, we find that the various mixtures give the following values for σ_{int}

Mixture	σ_{int}^{c} (Mev. -mb)	σ_{int}^{T} (Mev. -mb)
Serber	88.8	86.31
Inglis	106.5	102.0 Mev. -mb
Rosenfeld	106.5	102.0 Mev. -mb

A comparison between theory and experiment shows a serious discrepancy in σ_b between theory and experiment though there is a fairly good agreement between the two for σ_{int} for Inglis and Rosenfeld mixtures. The large disagreement between theory and experiment for σ_b may be explained due to the small rms value alpha particle given by Irving's wave functions.

As discussed above, the interpretation of Irving's small rms radius is not completely clear at present. McIntyre (56) has recently analysed the electron deuteron scattering successfully treating the proton as a spread out charge distribution and the neutron as a point charge. If we start from Hofstadter's measured rms radius of 1.61 f and following McIntyre subtract the contribution due to the proton's radius, we obtain an alpha rms radius of $\sqrt{(1.61)^2 - (0.77)^2} = 1.40$ f. (The subtraction of mean square radii is justified for Gaussian charge distributions, which are not in disagreement with Hofstadter's measurements).

For central forces $\sigma_b = \frac{4\pi^2}{3} \left(\frac{e^2}{\hbar c}\right) \frac{4}{3} (\overline{r^2})$. (5.53)

(See Eqns. (5.23) $\sigma_b \sim \langle u^2 \rangle_{oo}$ and (5.19) $\overline{r}^2 = \langle \frac{u^2}{4} + \frac{w^2}{2} \rangle_{oo} = \langle \frac{3u^2}{4} \rangle_{oo}$

Eqn. (5.53) is a good approximation for tensor forces. Using the experimental value $\sigma_b = 2.7$ in this equation we find $r_{rms} = 1.44$ f which supports Hofstadter's value of 1.40 f taking account of the proton size.

APPENDIX A

Spin Matrix Elements

We shall now evaluate the matrix elements for alpha particle 'D' state which is constructed in an operator form (See Irving (53)). These results are used in Chapter V in evaluation of $(Vr^2)_{oo}$. These operators act on the singlet spin state χ, so that the projection operator of the form $S = (1 - \vec{\sigma}_1 . \vec{\sigma}_2)(1 - \vec{\sigma}_3 . \vec{\sigma}_4)/16$ must be introduced to reduce the spin summations to the trace of an operator. Consider any two state operators Q_1 and Q_2 involving the spin vectors and acting on any two states represented by $Q_1\chi$ and $Q_2\chi$. If O is the operator for the interaction, the spin matrix element $\langle (Q_1\chi)^* \mid O \mid (Q_2\chi) \rangle$ is then given by

$$\mathrm{Spur}_{1,2,3,4} Q_1 O Q_2 S \tag{1}$$

the spur being taken in the spin-space of the particles 1, 2, 3, 4. In order to evaluate the spurs of the expressions arising from various matrix elements, we shall use the following results for any vectors $\vec{A}, \vec{B}, \vec{C}, \vec{D}$:

$$(\vec{\sigma}.\vec{A})(\vec{\sigma}.\vec{B}) = \vec{A}.\vec{B} + i\vec{\sigma}.(\vec{A} \times \vec{B})$$

$$\mathrm{Sp}(\vec{\sigma}.\vec{A}) = 0$$

$$\mathrm{Sp}(\vec{\sigma}.\vec{A})(\vec{\sigma}.\vec{B}) = 2(\vec{A}.\vec{B})$$

$$\mathrm{Sp}(\vec{\sigma}.\vec{A})(\vec{\sigma}.\vec{B})(\vec{\sigma}.\vec{C}) = 2i(\vec{A} \times \vec{B}).\vec{C}$$

$$\mathrm{Sp}(\vec{\sigma}.\vec{A})(\vec{\sigma}.\vec{B})(\vec{\sigma}.\vec{C})(\vec{\sigma}.\vec{D}) = 2\left[(\vec{A}.\vec{B})(\vec{C}.\vec{D}) - (\vec{A}.\vec{C})(\vec{B}.\vec{D}) + (\vec{A}.\vec{D})(\vec{B}.\vec{C})\right] \tag{2}$$

We shall illustrate the method of evaluating these spurs in a couple of cases and write the results of the rest. We shall first of all evaluate the spin matrix element $\langle {}^5D_o^{(1)*} \mid {}^5D_o^{(1)} \rangle$.

Writing the spin vectors Q_1 and Q_2 from Eqn. (5. 12) and using Eqn. (1), the matrix element reduces to the spur:

$$\mathrm{Sp}_{1,2,3,4}\left\{3(\vec{\sigma}_1\cdot\vec{\rho}_1)(\vec{\sigma}_3\cdot\vec{\rho}_2)+3(\vec{\sigma}_1\cdot\vec{\rho}_2)(\vec{\sigma}_3\cdot\vec{\rho}_1)-2(\vec{\rho}_1\cdot\vec{\rho}_2)(\vec{\sigma}_1\cdot\vec{\sigma}_3)\right\}$$
$$\left\{3(\vec{\sigma}_1\cdot\vec{\rho}_1)(\vec{\sigma}_3\cdot\vec{\rho}_2)+3(\vec{\sigma}_1\cdot\vec{\rho}_2)(\vec{\sigma}_3\cdot\vec{\rho}_1)-2(\vec{\rho}_1\cdot\vec{\rho}_2)(\vec{\sigma}_1\cdot\vec{\sigma}_3)\right\}$$
$$\left\{1-(\vec{\sigma}_1\cdot\vec{\sigma}_2)-(\vec{\sigma}_3\cdot\vec{\sigma}_4)+(\vec{\sigma}_1\cdot\vec{\sigma}_2)(\vec{\sigma}_3\cdot\vec{\sigma}_4)\right\}/16$$

Since spur of terms involving only one σ_i vanishes we see that only nine terms which include one from the last parenthesis do not vanish. We shall evaluate them one by one in the following.

(i) $(9/16)\,\mathrm{Sp}_{1,2,3,4}(\vec{\sigma}_1\cdot\vec{\rho}_1)(\vec{\sigma}_3\cdot\vec{\rho}_2)(\vec{\sigma}_1\cdot\vec{\rho}_1)(\vec{\sigma}_3\cdot\vec{\rho}_2)$

$=(9/4)\,\mathrm{Sp}_1(\vec{\sigma}_1\cdot\vec{\rho}_1)(\vec{\sigma}_1\cdot\vec{\rho}_1)\,\mathrm{Sp}_3(\vec{\sigma}_3\cdot\vec{\rho}_2)(\vec{\sigma}_3\cdot\vec{\rho}_2) = 9\,\rho_1^2\,\rho_2^2 .$

(ii) $(9/16)\,\mathrm{Sp}_{1,2,3,4}(\vec{\sigma}_1\cdot\vec{\rho}_1)(\vec{\sigma}_3\cdot\vec{\rho}_2)(\vec{\sigma}_1\cdot\vec{\rho}_2)(\vec{\sigma}_3\cdot\vec{\rho}_1)$

$=(9/4)\,\mathrm{Sp}_1(\vec{\sigma}_1\cdot\vec{\rho}_1)(\vec{\sigma}_1\cdot\vec{\rho}_2)\;\mathrm{Sp}_3(\vec{\sigma}_3\cdot\vec{\rho}_2)(\vec{\sigma}_3\cdot\vec{\rho}_1) = 9\,(\vec{\rho}_1\cdot\vec{\rho}_2)^2$

(iii) $-(6/16)(\vec{\rho}_1\cdot\vec{\rho}_2)\;\mathrm{Sp}_{1,2,3,4}(\vec{\sigma}_1\cdot\vec{\rho}_1)(\vec{\sigma}_3\cdot\vec{\rho}_2)(\vec{\sigma}_1\cdot\vec{\sigma}_3)$

$=-(6/4)(\vec{\rho}_1\cdot\vec{\rho}_2)\,\mathrm{Sp}_{1,3}(\vec{\sigma}_1\cdot\vec{\rho}_1)(\vec{\sigma}_3\cdot\vec{\rho}_2)(\vec{\sigma}_1\cdot\vec{\sigma}_3)$

$=-(6/2)(\vec{\rho}_1\cdot\vec{\rho}_2)\,\mathrm{Sp}_3(\vec{\sigma}_3\cdot\vec{\rho}_1)(\vec{\sigma}_3\cdot\vec{\rho}_2) = -6\,(\vec{\rho}_1\cdot\vec{\rho}_2)^2$

(iv) $(9/16)\,\mathrm{Sp}_{1,2,3,4}(\vec{\sigma}_1\cdot\vec{\rho}_2)(\vec{\sigma}_3\cdot\vec{\rho}_1)(\vec{\sigma}_1\cdot\vec{\rho}_1)(\vec{\sigma}_3\cdot\vec{\rho}_2) = 9\,(\vec{\rho}_1\cdot\vec{\rho}_2)^2$

(v) $(9/16)\,\mathrm{Sp}_{1,2,3,4}(\vec{\sigma}_1\cdot\vec{\rho}_2)(\vec{\sigma}_3\cdot\vec{\rho}_1)(\vec{\sigma}_1\cdot\vec{\rho}_2)(\vec{\sigma}_3\cdot\vec{\rho}_1) = 9\,\rho_1^2\,\rho_2^2$

(vi) $-(6/16)(\vec{\rho}_1\cdot\vec{\rho}_2)\,\mathrm{Sp}_{1,2,3,4}(\vec{\sigma}_1\cdot\vec{\rho}_2)(\vec{\sigma}_3\cdot\vec{\rho}_1)(\vec{\sigma}_1\cdot\vec{\sigma}_3) = -6\,(\vec{\rho}_1\cdot\vec{\rho}_2)^2$

(vii) $-(6/16)(\vec{\rho}_1\cdot\vec{\rho}_2)\,\mathrm{Sp}_{1,2,3,4}(\vec{\sigma}_1\cdot\vec{\sigma}_3)(\vec{\sigma}_1\cdot\vec{\rho}_1)(\vec{\sigma}_3\cdot\vec{\rho}_2) = -6\,(\vec{\rho}_1\cdot\vec{\rho}_2)^2$

(viii) $-(6/16)(\vec{\rho}_1\cdot\vec{\rho}_2)\,\mathrm{Sp}_{1,2,3,4}(\vec{\sigma}_1\cdot\vec{\sigma}_3)(\vec{\sigma}_1\cdot\vec{\rho}_1)(\vec{\sigma}_3\cdot\vec{\rho}_1) = -6\,(\vec{\rho}_1\cdot\vec{\rho}_2)^2$

(ix) $(4/16\,(\vec{\rho}_1\cdot\vec{\rho}_2)^2\,\mathrm{Sp}_{1,2,3,4}(\vec{\sigma}_1\cdot\vec{\sigma}_3)(\vec{\sigma}_1\cdot\vec{\sigma}_3)$

$=(\vec{\rho}_1\cdot\vec{\rho}_2)^2\,\mathrm{Sp}_{1,3}(\vec{\sigma}_1\cdot\vec{\sigma}_3)(\vec{\sigma}_1\cdot\vec{\sigma}_3) = 2\,(\vec{\rho}_1\cdot\vec{\rho}_2)^2\,\mathrm{Sp}_3\sigma_3^2 = 12\,(\rho_1\cdot\rho_2)^2$

Combining all the results we get in agreement with Irving

$$\left\langle {}^5D_0^{(1)*} \,\middle|\, {}^5D_0^{(1)} \right\rangle = 18\,\rho_1^2\,\rho_2^2 + 6\,(\vec{\rho}_1\cdot\vec{\rho}_2)^2 .$$

It is of course obvious that $\left\langle {}^1S_o^* \middle| {}^1S_o \right\rangle = 1$. As a next step, we shall evaluate the matrix element $\left\langle {}^1S_o^* \middle| S_{14} \middle| {}^5D_o^{(1)} \right\rangle$ and show how other similar matrix elements can be obtained from it. Again, writing the spin vectors and using Eqn. (1) we find that we have to evaluate the spur of the following expression:

$$Sp_{1,2,3,4}\left\{\frac{3(\vec{\sigma}_1 . \vec{r}_{14})(\vec{\sigma}_4 . \vec{r}_{14})}{r_{14}^2} - (\vec{\sigma}_1 . \vec{\sigma}_4)\right\}$$
$$\left\{3(\vec{\sigma}_1 . \vec{\rho}_1)(\vec{\sigma}_3 . \vec{\rho}_2) + 3(\vec{\sigma}_1 . \vec{\rho}_2)(\vec{\sigma}_3 . \vec{\rho}_1) - 2(\vec{\rho}_1 . \vec{\rho}_2)(\vec{\sigma}_1 . \vec{\sigma}_3)\right\}$$
$$\left\{1 - (\vec{\sigma}_1 . \vec{\sigma}_2) - (\vec{\sigma}_3 . \vec{\sigma}_4) + (\vec{\sigma}_1 . \vec{\sigma}_2)(\vec{\sigma}_3 . \vec{\sigma}_4)\right\} \Big/ 16$$

In this spur, there are only six non-vanishing terms which are evaluated below.

(i) $-(9/16r_{14}^2)\, Sp_{1,2,3,4}\, (\vec{\sigma}_1 . \vec{r}_{14})(\vec{\sigma}_4 . \vec{r}_{14})(\vec{\sigma}_1 . \vec{\rho}_1)(\vec{\sigma}_3 . \vec{\rho}_2)(\vec{\sigma}_3 . \vec{\sigma}_4)$

$= -(9/8r_{14}^2)\, Sp_1\, (\vec{\sigma}_1 . \vec{r}_{14})(\vec{\sigma}_1 . \vec{\rho}_1)\, Sp_{3,4}(\vec{\sigma}_4 . \vec{r}_{14})(\vec{\sigma}_3 . \vec{\rho}_2)(\vec{\sigma}_3 . \vec{\sigma}_4)$

$= -(9/2)\, \dfrac{(\vec{\rho}_1 . \vec{r}_{14})}{r_{14}^2}\; Sp_4\, (\vec{\sigma}_4 . \vec{\rho}_2)(\vec{\sigma}_4 . \vec{r}_{14}) = -\dfrac{9(\vec{\rho}_1 . \vec{r}_{14})(\vec{\rho}_2 . \vec{r}_{14})}{r_{14}^2}$

(ii) $(3/16)\, Sp_{1,2,3,4}\, (\vec{\sigma}_1 . \vec{\sigma}_4)(\vec{\sigma}_1 . \vec{\rho}_1)(\vec{\sigma}_3 . \vec{\rho}_2)(\vec{\sigma}_3 . \vec{\sigma}_4)$

$= (3/4)\, Sp_{3,4}\, (\vec{\sigma}_4 . \vec{\rho}_1)(\vec{\sigma}_3 . \vec{\rho}_2)(\vec{\sigma}_3 . \vec{\sigma}_4)$

$= (3/2)\, Sp_4\, (\vec{\sigma}_4 . \vec{\rho}_1)(\vec{\sigma}_4 . \vec{\rho}_2) = 3\,(\vec{\rho}_1 . \vec{\rho}_2)$

(iii) $-(9/16r_{14}^2)\, Sp_{1,2,3,4}\, (\vec{\sigma}_1 . \vec{r}_{14})(\vec{\sigma}_4 . \vec{r}_{14})(\vec{\sigma}_1 . \vec{\rho}_2)(\vec{\sigma}_3 . \vec{\rho}_1)(\vec{\sigma}_3 . \vec{\sigma}_4)$

$= -9\,(\vec{\rho}_1 . \vec{r}_{14})(\vec{\rho}_2 . \vec{r}_{14})/r_{14}^2$

(iv) $(3/16)\, Sp_{1,2,3,4}\, (\vec{\sigma}_1 . \vec{\sigma}_4)(\vec{\sigma}_1 . \vec{\rho}_2)(\vec{\sigma}_3 . \vec{\rho}_1)(\vec{\sigma}_3 . \vec{\sigma}_4) = 3\,(\vec{\rho}_1 . \vec{\rho}_2)$

(v) $\left(6(\vec{\rho}_1 . \vec{\rho}_2)/16r_{14}^2\right) Sp_{1,2,3,4}\, (\vec{\sigma}_1 . \vec{r}_{14})(\vec{\sigma}_4 . \vec{r}_{14})(\vec{\sigma}_1 . \vec{\sigma}_3)(\vec{\sigma}_3 . \vec{\sigma}_4) = 12(\vec{\rho}_1 . \vec{\rho}_2)$

(vi) $-(2/16)\,(\vec{\rho}_1 . \vec{\rho}_2)\, Sp_{1,2,3,4}(\vec{\sigma}_1 . \vec{\sigma}_3)(\vec{\sigma}_1 . \vec{\sigma}_4)(\vec{\sigma}_3 . \vec{\sigma}_4) = -12\,(\vec{\rho}_1 . \vec{\rho}_2)$

Combining all the results, we get

$$\left\langle {}^1S_o^* \middle| S_{14} \middle| {}^5D_o^{(1)} \right\rangle = -18\,\frac{(\vec{\rho}_1 . \vec{r}_{14})(\vec{\rho}_2 . \vec{r}_{14})}{r_{14}^2} + 6\,(\vec{\rho}_1 . \vec{\rho}_2) \qquad (3)$$

It is now very easy to calculate the matrix element $\langle {}^1S_0^* | S_{13} | {}^5D_0^{(1)} \rangle$. We have simply to study the effect of replacing 3 by 4 in the wave function. Such an alteration changes the sign of ${}^1S_0$ and ${}^5D_0$. and since $\vec{\rho}_2 = \vec{r}_4 - \vec{r}_3$ therefore $\vec{\rho}_2$ will also change sign. Thus there will be an overall change of sign in the right side of Eqn. (3) and r_{14} will be replaced by r_{13}.

Therefore

$$\langle {}^1S_0^* | S_{13} | {}^5D_0^{(1)} \rangle = \left\{ 18 \frac{(\vec{r}_{13} \cdot \vec{\rho}_1)(\vec{r}_{13} \cdot \vec{\rho}_2)}{r_{13}^2} - 6(\vec{\rho}_1 \cdot \vec{\rho}_2) \right\}$$

in agreement with Irving's Appendix A, Eqn. (4)

Similarly

$$\langle {}^1S_0^* | S_{23} | {}^5D_0^{(1)} \rangle = \left\{ -18 \frac{(\vec{r}_{23} \cdot \vec{\rho}_1)(\vec{r}_{23} \cdot \vec{\rho}_2)}{r_{23}^2} + 6(\vec{\rho}_1 \cdot \vec{\rho}_2) \right\}$$

and $$\langle {}^1S_0^* | S_{24} | {}^5D_0^{(1)} \rangle = \left\{ 18 \frac{(\vec{r}_{24} \cdot \vec{\rho}_1)(\vec{r}_{24} \cdot \vec{\rho}_2)}{r_{24}^2} - 6(\vec{\rho}_1 \cdot \vec{\rho}_2) \right\}$$

$$\langle {}^1S_0^* | S_{24} | {}^5D_0^{(1)} \rangle = \langle {}^1S_0^* | S_{34} | {}^5D_0^{(1)} \rangle = 0$$

We shall in the following state the results of the rest of the spin matrix elements.

$$\langle {}^5D_0^{*(1)} | S_{12} | {}^5D_0^{(1)} \rangle = \langle {}^5D_0^{*(1)} | S_{34} | {}^5D_0^{(1)} \rangle = -\left\{ 18 \rho_1^2 \rho_2^2 + 30 (\vec{\rho}_1 \cdot \vec{\rho}_2)^2 \right\}$$

$$\langle {}^5D_0^{*(1)} | S_{13} | {}^5D_0^{(1)} \rangle = \left\{ 42 (\vec{\rho}_1 \cdot \vec{\rho}_2)^2 - 18 \rho_1^2 \rho_2^2 - 72 (\vec{r}_{13} \cdot \vec{\rho}_1)(\vec{r}_{13} \cdot \vec{\rho}_2)(\vec{\rho}_1 \cdot \vec{\rho}_2)/r_{13}^2 + 54 \frac{[(\vec{\rho}_1 \times \vec{r}_{13}) \cdot \vec{\rho}_2]^2}{r_{13}^2} \right\}$$

$$\langle {}^5D_0^{*(1)} | S_{14} | {}^5D_0^{(1)} \rangle = \left\{ 42 (\vec{\rho}_1 \cdot \vec{\rho}_2)^2 - 18 \rho_1^2 \rho_2^2 - 72 (\vec{r}_{14} \cdot \vec{\rho}_1)(\vec{r}_{14} \cdot \vec{\rho}_2)(\vec{\rho}_1 \cdot \vec{\rho}_2)/r_{14}^2 + 54 \frac{[(\vec{\rho}_1 \times \vec{r}_{14}) \cdot \vec{\rho}_2]^2}{r_{14}^2} \right\}$$

$$\left\langle {}^5D_o^{(1)*} \right| S_{24} \left| {}^5D_o^{(1)} \right\rangle = \Big\{ 42\,(\vec{\rho}_1 \cdot \vec{\rho}_2)^2 - 18\,\rho_1^2 \rho_2^2 - 72\,\frac{(\vec{r}_{24} \cdot \vec{\rho}_1)\,(\vec{r}_{24} \cdot \vec{\rho}_2)\,(\vec{\rho}_1 \cdot \vec{\rho}_2)}{r_{24}^2}$$

$$+\, 54\,\frac{\left[(\vec{\rho}_1 \times \vec{r}_{24}) \cdot \vec{\rho}_2\right]^2}{r_{24}^2} \Big\}$$

$$\left\langle {}^5D_o^{(1)*} \right| S_{23} \left| {}^5D_o^{(1)} \right\rangle = \Big\{ 42\,(\vec{\rho}_1 \cdot \vec{\rho}_2)^2 - 18\,\rho_1^2 \rho_2^2 - 72\,\frac{(\vec{r}_{23} \cdot \vec{\rho}_1)\,(\vec{r}_{23} \cdot \vec{\rho}_2)\,(\vec{\rho}_1 \cdot \vec{\rho}_2)}{r_{23}^2}$$

$$+\, 54\,\frac{\left[(\vec{\rho}_1 \times \vec{r}_{23}) \cdot \vec{\rho}_2\right]^2}{r_{23}^2} \Big\}$$

APPENDIX B

The following integrals appear in the calculations.

$$\int_0^1 \frac{(1-u^2)^{1/2} u^3 du}{(1 + c u)^7} = \frac{1}{720} \left\{ \frac{96 + 741c^2 + 120c^4 - 12c^6}{(1 - c^2)^5} - \frac{315c(1 + 2c^2)\cos^{-1} c}{(1 - c^2)^{11/2}} \right\}$$

$$\int_0^1 \frac{u^5(1-u^2)^2 du}{(c + u)^{12}} = \frac{231\, c^3 + 159c^2 + 45c + 5}{13860\, c^6\, (c + 1)^9}$$ (See also Irving 53)

$$\int_0^1 \frac{u^3(1-u^2)^2 du}{(c + u)^{10}} = \frac{21c^3 + 19c^2 + 7c + 1}{504c^6(c + 1)^7}$$ (See also Irving 53)

$$\int_0^1 \frac{(1-u^2)^3\, u^5 du}{(c + u)^{14}} = \frac{3003c^4 + 9630c^3 + 4710c^2 + 350c + 35}{360360\, c^8\, (c + 1)^{10}}$$

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